Ministry of Higher Education and Research

Badji Mokhtar University - Annaba
Faculty of Earth Sciences
Department of Architecture

TEACHER: SEDDIKI ZAHIRA
Senior lecturer “B”

Descriptive geometry
Course of lectures

F i r s t s e m e s t e r

For students of architecture
Licence and engineering first year
2023-2024

Dear Students,

First, I extend my sincere appreciation to my old students, for their dedication and commitment to learning in this course. Your active participation and thoughtful engagement during our lectures have contributed immensely to the overall classroom experience.

It is truly inspiring to witness the intellectual curiosity and passion for learning descriptive geometry. I am grateful for the opportunity to guide you and to share my knowledge and expertise with such motivated groups of students four years ago.

As we continue our exploration of geometry and architecture, I encourage all students to remain proactive in your studies and to approach each lesson with an open mind and a willingness to learn. Together, we can create a dynamic and enriching learning environment that fosters growth and development.

Warm regards,

Summary

Course 01 : Methods of projection & Representation of the point in space……… P 4-15

1 Introduction : 6

2 **Methods of projection :** 7

2.1 Orthographic (parallel) projection 7

2.2 perspective projection 8

3 Geometric space : 8

4 Representation of the point 11

Course 02 : Reflection On Octants & Point Representation………………….... P16-22

1 Introduction : 17

2 The coordinate of point in geometry descriptive : 18

3 Octant and quadrant: 18

3.1 Quadrant : 18

3.2 Octant 20

Course 03 : The Line And Notable Lines……………………………………… P 23-30

1 LINE : 24

2 Line projecting on the three projection planes : 25

3 Remarkable Lines: 26

- **THE VERTICAL LINE** 27
- **THE UPRIGHT(standing) LINE :** 27
- **THE HORIZONTAL LINE** 28
- **FRONTAL LINE :** 28
- **PROFIL LINE :** 29
- **PARALLEL LINE to two planes of projection:** 30

Course 04 : The piercing points of a line………………………………………. P 31-39

1 PIERCING POINTS : 31

2 Features of Points of Piercing: 31

3 Horizontal and Frontal Traces in the Case of Two Projection Planes 32

PH (x,0) 33

PF(0,y) 33

4 Horizontal, Frontal, and Profile Traces in the Case of Three Projection Planes: 35

PH (x,y,0) 35

PF (0,y,Z) 35

Pp (X, 0, Z) 36

Course 05 : Parallel and intersect lines…………………………………………… P 40-44

1 **Position of line in space:** 40

.1.1 Parallel lines: 40

.1.2 intersect lines: 42

.1.3 Skew lines: 43

Course 06: A plane and remarkable planes……………………………………… P45-58

1 **Determination of plane:** 45

.1.1 Plane traces: 47

.1.2 Remarkable planes 49

Course 07: Drawing transformation in geometry……………………………… P59-73

1 **Introduction** 59

2 **Changing the projection plane (or ground-line) / (Auxiliary View Method) :** 60

3 **Methods of rotation** 66

4 **Rabbatement:** 69

5 **Rotation around plane traces (or coinciding method)** 71

Course 08: plane surfaces (polyhedrons) and representation of circle / cone / cylinder / sphere……………………………………………………………………… P 74-87

1 **Defenition polyhedrons:** 74

2 **Prisms, Prismoids, and Pyramids :** 75

3 **Representation of cone / cylinder / sphere** 78

4 **Intersecting polyhedron and plane:** 81

5 **Intersecting polyhedron and line:** 82

6 **Intersecting of two polyhedrons:** 83

Course 09 : introduction to tree-dimensions geometry and axonometric projection…………………………………………………………………….. P98-99

1 **Understanding Three-Dimensional Space:** 89

2 **Demonstrating how to represent three-dimensional objects on a two-dimensional plane using projection techniques.** 93

3 **solid modeling techniques for representing complex 3D objects :** 96

Course 10: generalities about perspective…………………………………...... P 100-114

1 **Introduction:** 102

Course 11: application of the theory of shadows in monge projection and perspective and axonometry…………………………………………………………….. P115-129

1 **Introduction :** 119

2 **Definition of Shadows :** 119

3 **Laws of light and shadow projection :** 119

4 Projection of a Shadow of Point A (Application of the theory of shadow in monge projection): 121

5 Properties of Shadows :........ 126

Course N°01 : Methods of projection & Representation of the point in space

1 Introduction :

Geometry is that branch of mathematics which treats of spatial magnitudes and Examine the configuration of spatial forms. The word spatial relates to space, and space is the plenum or void in which all material bodies have their being.

Idealised 3-space is the abstract representational notion we have of the void or plenum in which we live and in which everything known to us has its being.

Descriptive geometry can be described as the method of accurately depicting objects through drawing and visually solving all issues pertaining to their shape and positioning.[1]

Descriptive geometry offers the theoretical foundation for technical drawing. In a broad sense, an object or structure can be viewed as a composition of basic geometric elements, including lines, points, planes, and shapes.

[1] The father and inventor of this discipline is Gaspard Monge, born in Beaune (France) on May 9, 1746. At the age of 17, after creating a plan of his hometown, he caught the attention of the staff at the engineering school in Mézières, and he was hired as a draftsman. His talents as a geometer quickly became evident when he drew plans for new fortifications considered impregnable at the time. His rapid and elegant graphical methods would be the foundation of what is now known as descriptive geometry.

2 Methods of projection :

In descriptive geometry and technical drawing, projection methods are essential tools for representing three-dimensional objects on a two-dimensional surface. Orthographic projection and perspective projection are the two primary categories of projection techniques.

2.1 Orthographic (parallel) projection

Multiview Projection typically involves utilizing three views – front, top, and side – in orthographic projection to accurately depict the shape and proportions of an object from different perspectives. On the other hand, Axonometric Projection maintains parallel lines in all three dimensions at consistent angles. This technique includes variations such as isometric, dimetric, and trimetric projections**. For example, technical representations often use isometric projection, where three equally foreshortened axes are depicted.**

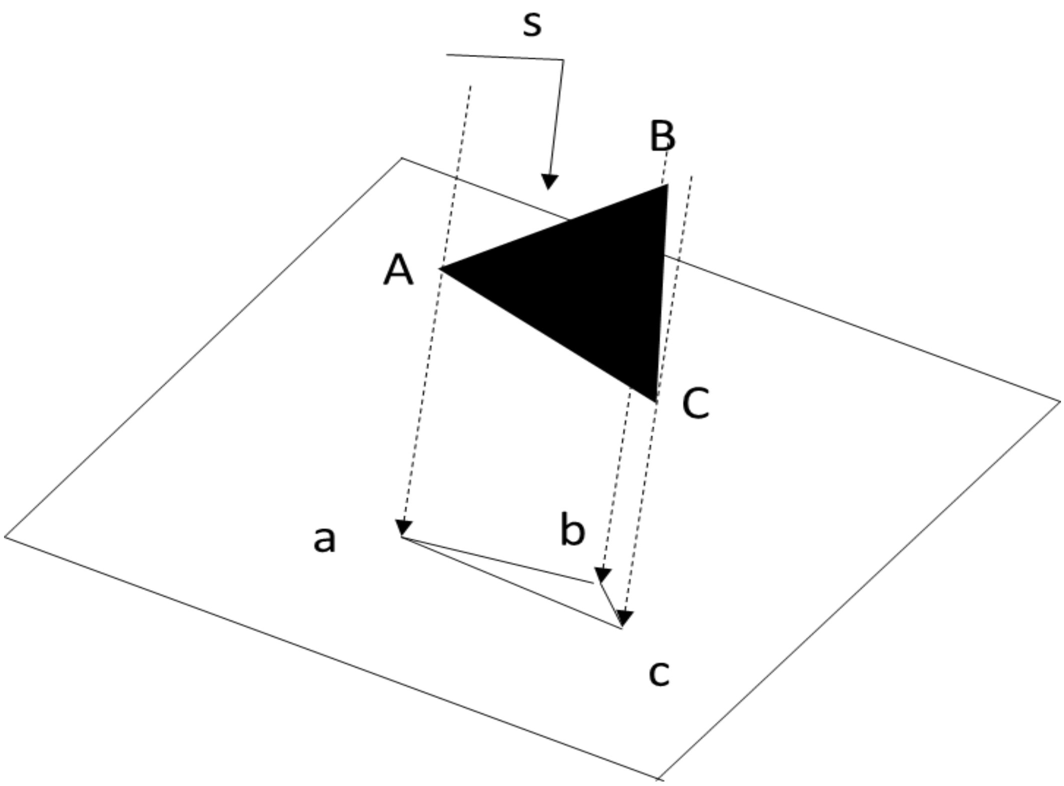


Figure 1 : source author

2.2 Perspective projection

One-Point Perspective: Using this technique, every parallel line converges to a single horizonal vanishing point. It works well for depicting items as seen from the front.

Two-Point Perspective: This technique use two vanishing points to show objects as seen from an angle.

Three-Point Perspective: This technique allows for the representation of objects with vertical sides or when viewed from an elevated or lowered perspective. It incorporates a third vanishing point.

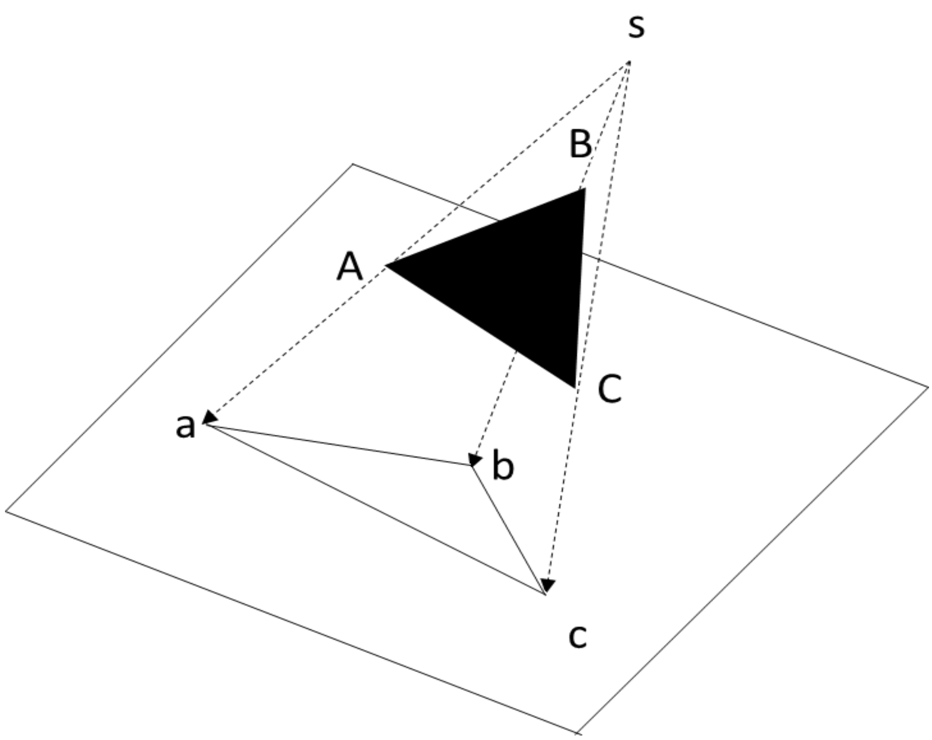


Figure 2 : source author

3 Geometric space :

To define geometric space, we have introduced two characteristics of this geometric space in its purely mathematical dimension. These components in interaction within a given context are as follows:

- A non-real and non-local space as the support for immateriality with an infinity of possibilities of objects, points, lines, etc., non-concrete and intangible.
- Defined by three axes or two axes such as: $\bar{X}$, $\bar{Y}$, $\bar{Z}$

In other words: "This space possesses the following properties: it is infinite, continuous, ***has three dimensions***, all its elements (points) are identical to each other, and all lines passing through the same point are identical to each other." (*espace géométrique* | *Lexique de mathématique*, 2016)

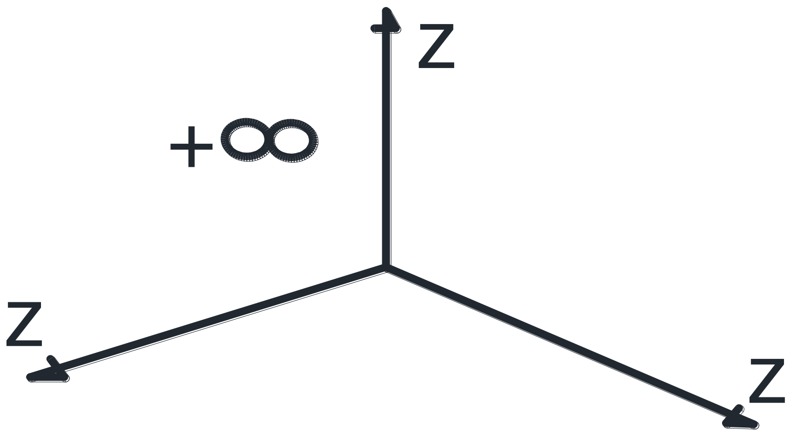


Figure 3 : source author

The intersection of these three axes results in three planes, which are: the horizontal, frontal, and profile planes."

1- METHOD OF ORTHOGRAPHIC PROJECTION :

When the projection angles deviate from perpendicular to the plane, the parallel projection method is known as oblique projection. If the angles are perpendicular, it is termed orthogonal projection. In orthogonal projection, any point in space that is perpendicular to a plane and intersects with a straight line is considered to be orthogonally projected onto the plane, with the perpendicular line being referred to as the projector."

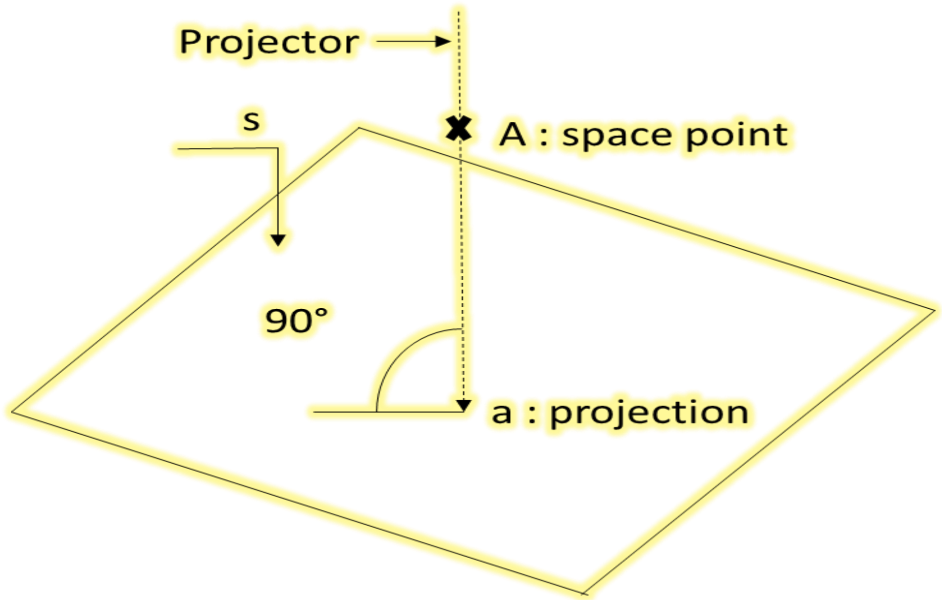


Figure 4 : source author

If the plane of the drawing space is to contain the origin 0, then in first angle projection the three mutually perpendicular picture planes that form the space of projection.

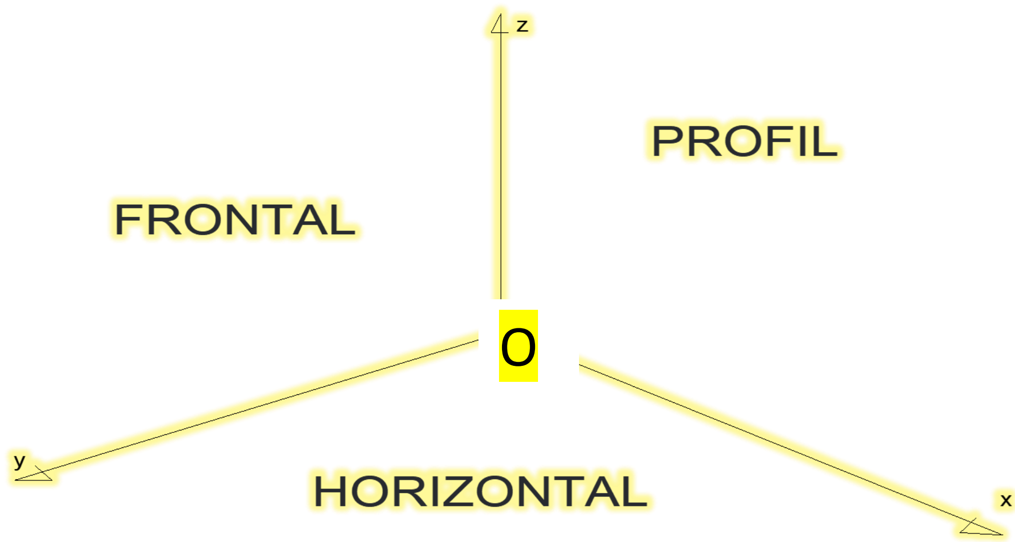


Figure 5 : source author

4 Representation of the point

Two main methods are used:

<u>A) point A in space is located relative to two planes using projections onto two planes and its distance from the plane. This is the method of Crate Geometry."</u>

To represent an arbitrary point A in space, it is necessary to project it onto two planes, denoted as H and F, which are respectively the horizontal plane and the frontal plane. Subsequently, to draw on a floor, which is the same drawing plane, one mentally rotates the frontal plane onto the extension of the horizontal plane, forming the intersection of the two planes called the ground line. In technical drawings, the horizontal projection of any point in space is represented by a lowercase letter such as 'a' or 'a', or alternatively denoted as (a, a').

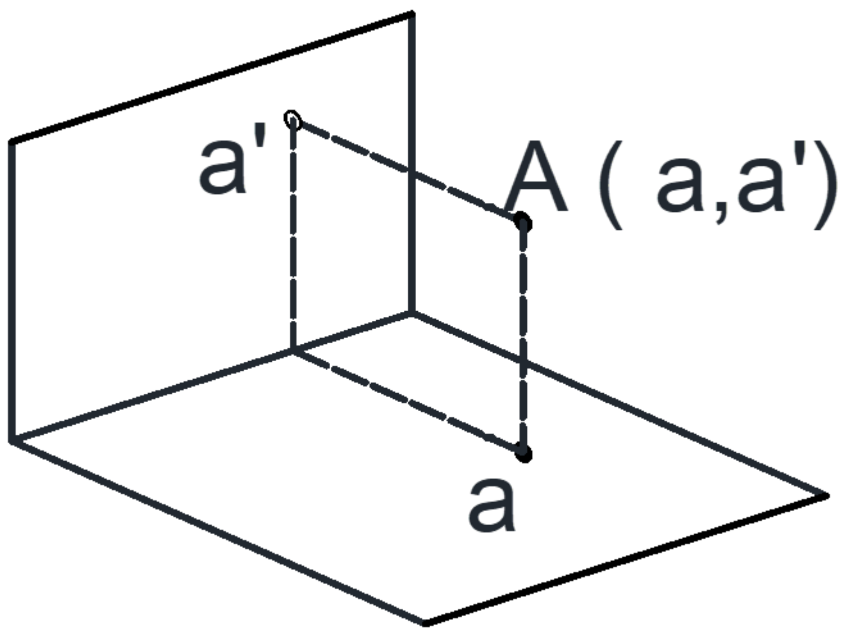


Figure 6 : source author

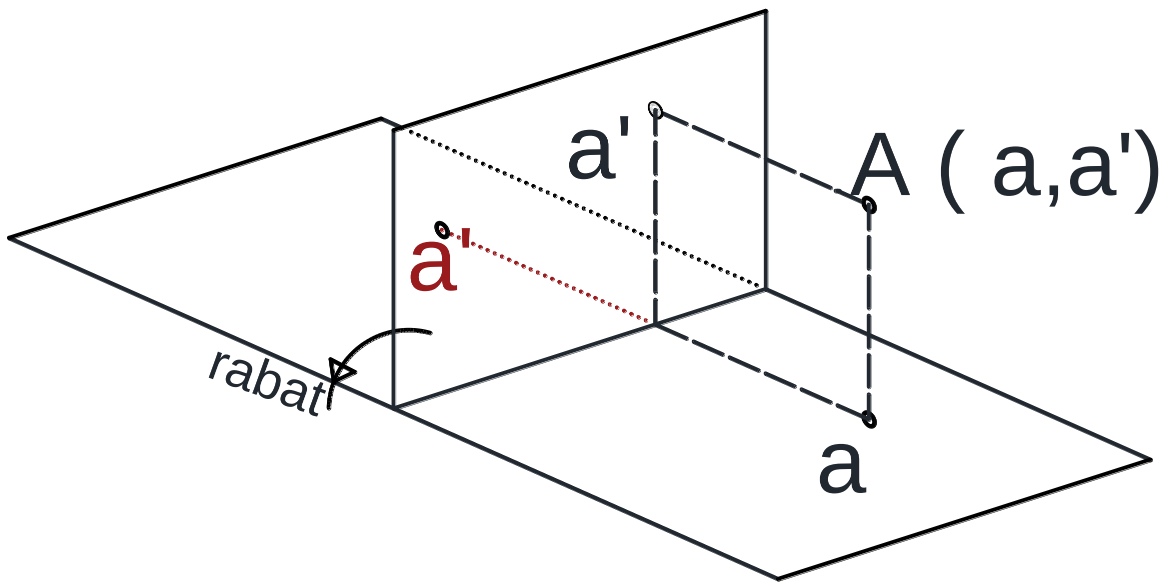


Figure 7 : **source author**

To ensure that two points on a draft are projections of the same point in space, the required condition is that the line connecting these two points, denoted as a and à, must be perpendicular to the horizon line. This condition arises from the preceding arguments, demonstrating that when the line aà connecting these points is orthogonal to the horizon line, they represent projections of the same point, denoted as A, in space.

This specific condition ensures that the projection of points onto the plane is performed perpendicularly to the horizon line, crucial for ensuring the accuracy of the graphical representation. Thus, when this perpendicular relationship is maintained between the line connecting the points on the draft and the horizon line, it can be concluded that these points are projections of a single three-dimensional point in space.

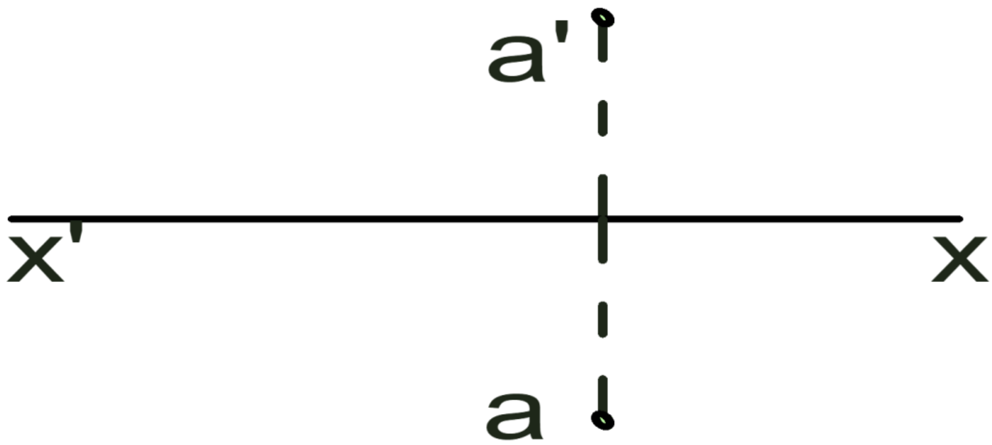


Figure 8 : **source author**

b) A point M in space is located using its orthogonal projections on three rectangular planes.

This is the method of Descriptive Geometry with three projection planes, or Monge's Geometry, named after its inventor.

A (x, y, z) / (x: abscissa, y: ordinate, z: applicate)

Theorem: Three points a, a', and a'' from a drawing can be considered as horizontal, frontal, and profile projections of a point A in space provided that they are located at the intersections (90°) of three reference lines.

Writing Convention: The three projections of a point are always indicated by the same lowercase letter. For exemple : The horizontal projection of a point A is denoted as m. The frontal projection is denoted by the same letter a'. The profile projection is denoted by the same letter a''.

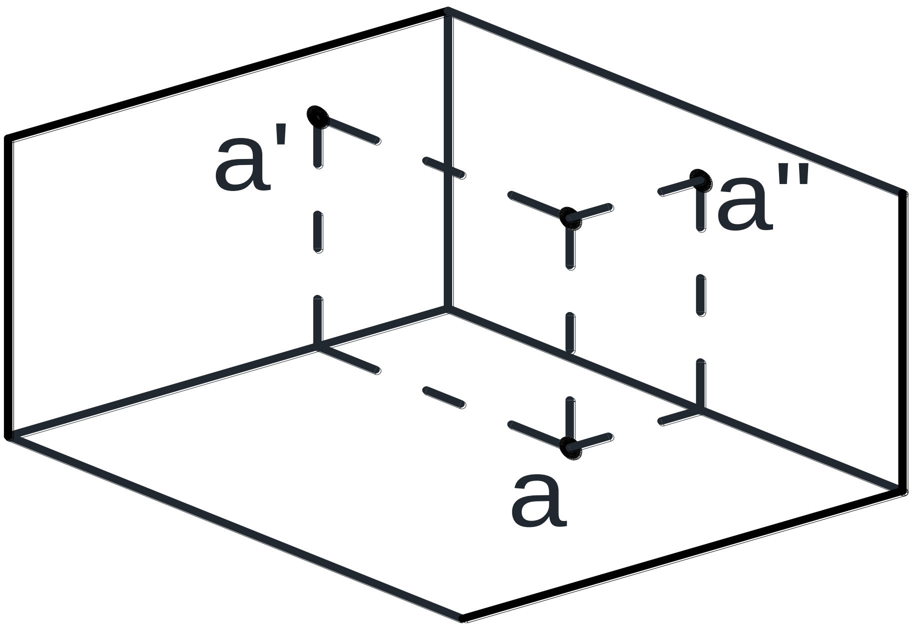


Figure 9 : **source author**

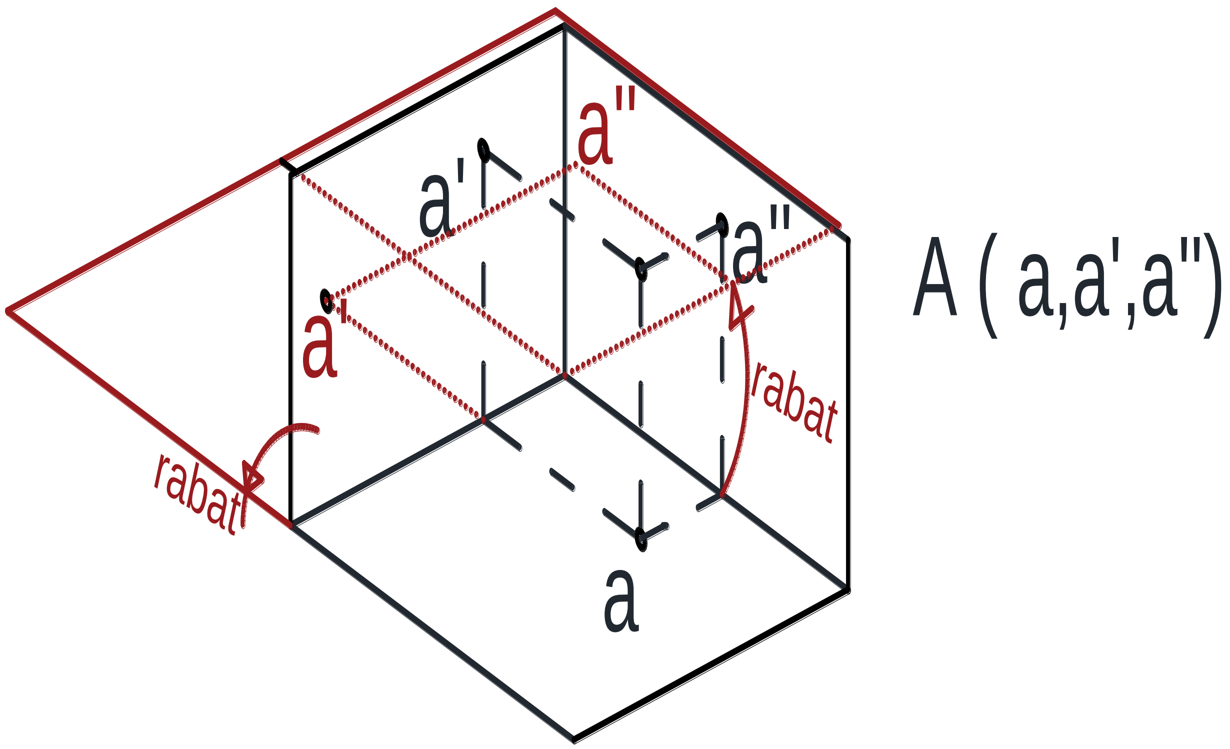


Figure 10 : source author

The process of determining the projections of a point in space involves dropping a perpendicular line from that point to three planes: horizontal, frontal, and profile. Specifically, starting from point A, a perpendicular line Aa, referred to as the projecting line, is extended to the horizontal plane. The point where this line meets the horizontal plane, denoted as a, represents the horizontal projection of the original point A.

Similarly, another perpendicular line, Aà, is drawn from point A to the frontal plane. The intersection point of this line with the frontal plane, labeled as a', indicates the frontal projection of the initial point A.

This same principle is applied to obtain the profile projection of the point A. By extending a perpendicular line A" from point A to the profile plane, the point of intersection, denoted as a", signifies the profile projection of the original point in space.

In summary, the projections on the three planes are determined by projecting perpendicular lines from the point to each respective plane.

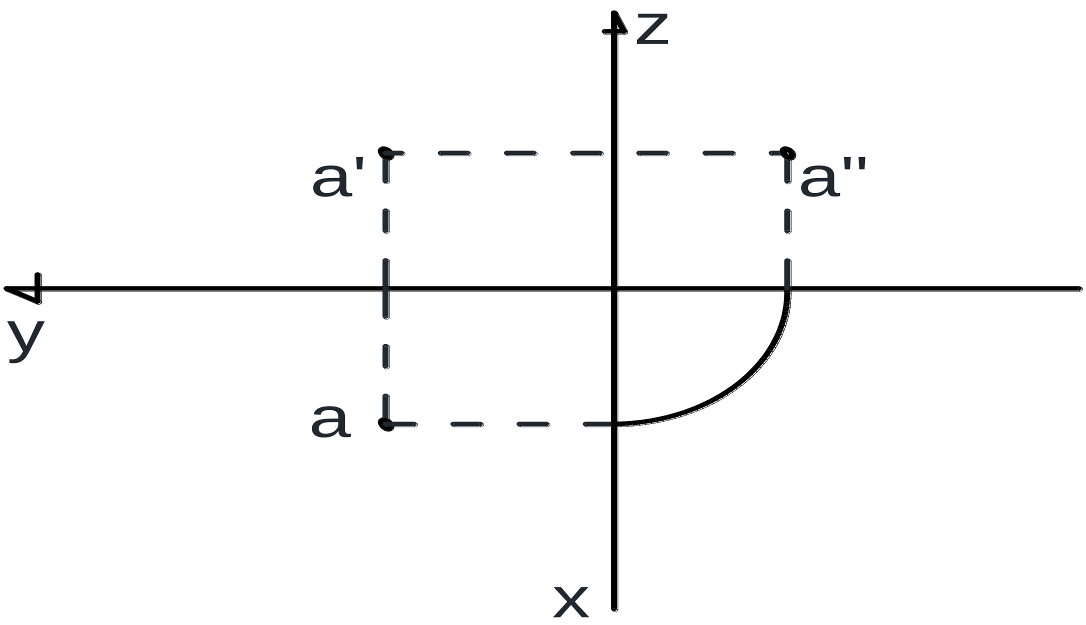


Figure 11 : source author

To depict three projections of an object on a single plane, adjustments need to be made to the positions of these planes. This adjustment involves rotating either projection plane around the ground line, OX and OY, until it aligns with the plane F. This alignment occurs when the back portion of the H plane rests upon the upper portion of F plane. And made the rebattement to the profile plane.

COURSE N°02 : Reflection on octants & Point representation

1 Introduction :

Coordinate Systems, generely, start by presenting the notion of coordinate systems employed in spatial geometry, including Cartesian coordinates (x, y, z), cylindrical coordinates (r, θ, z), and spherical coordinates (ρ, θ, φ). Clarify how these systems are employed to pinpoint locations in three-dimensional space.

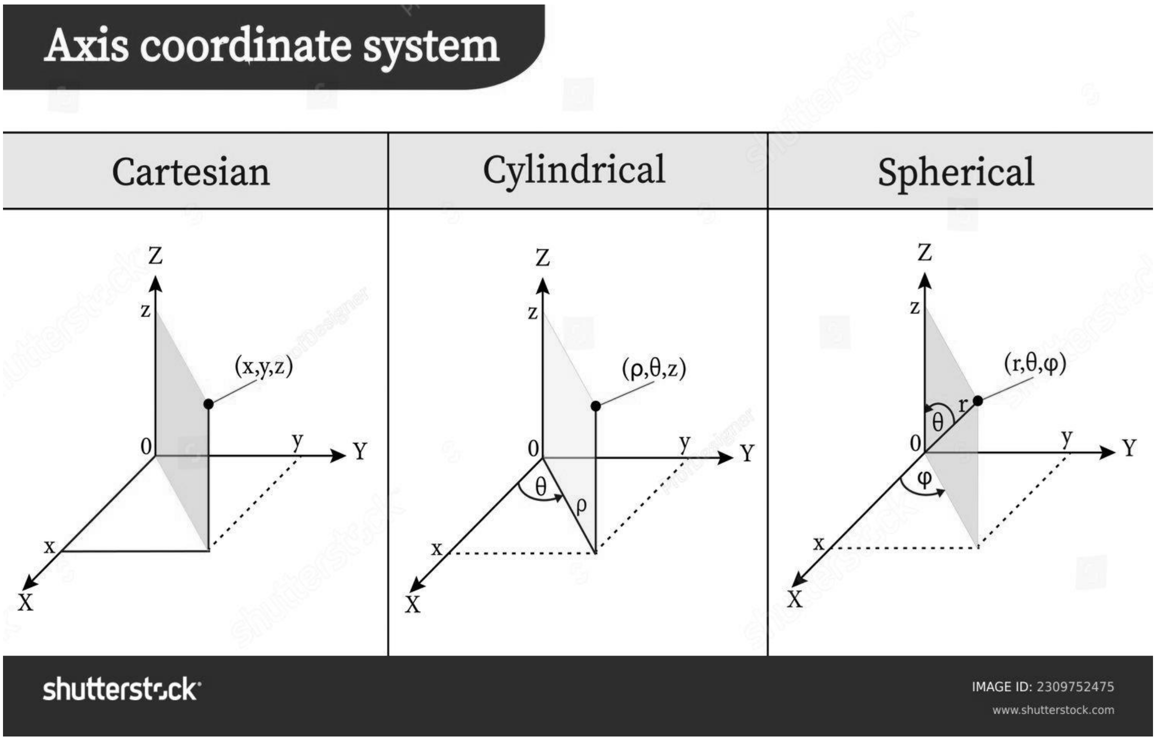


Figure 12 : Source : https://www.shutterstock.com/search/cartesian-coordinate-system

2 The coordinate of point in geometry descriptive :

The point's location in descriptive geometry is determined by its cartisian coordinates. In descriptive geometry, the drawing of objects employs coordinate representation instead of isometric display for specific purposes.

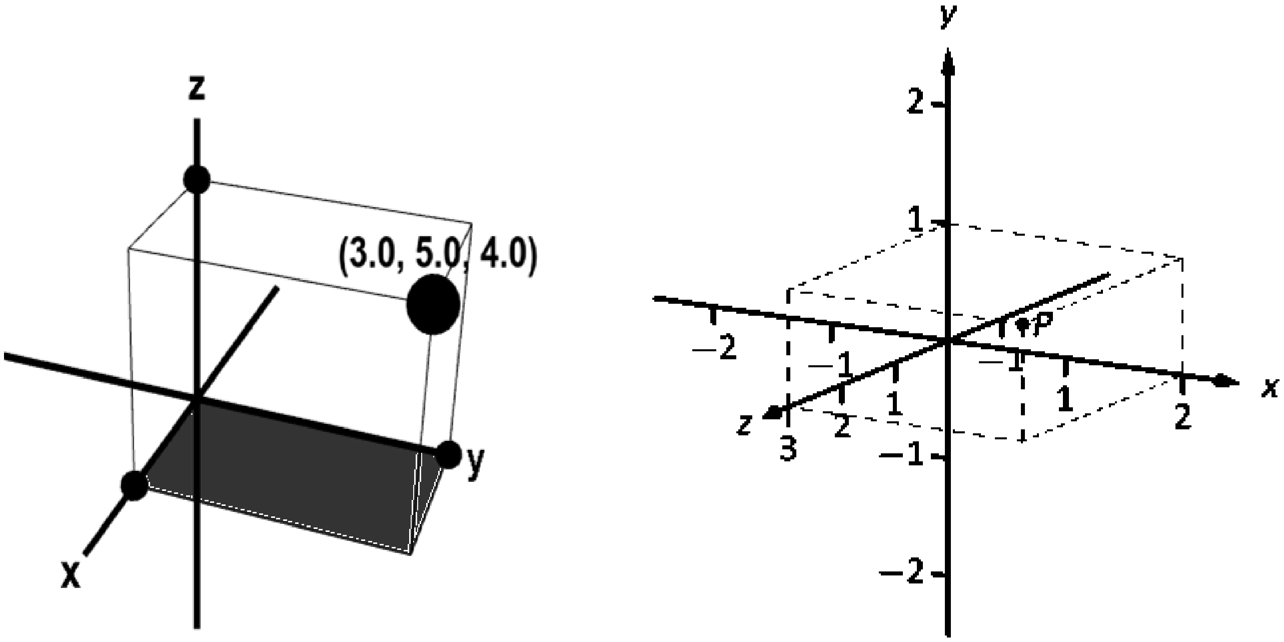


Figure 13 : Source : https://mathinsight.org/cartesian_coordinates

3 Octant and quadrant:

It is a geometric space positioned in relation to our similar space, defined either by the intersection of two planes or by three planes.

3.1 Quadrant :

A geometric area or segment that represents one-fourth (1/4) of a circle is called a quadrant. Within the domain of Cartesian coordinates, which is commonly utilized in fields such as mathematics, physics, and engineering, a quadrant represents one of the four segments formed by the point where the x and y axes intersect on the coordinate plane.

- L'intersection of two planes :

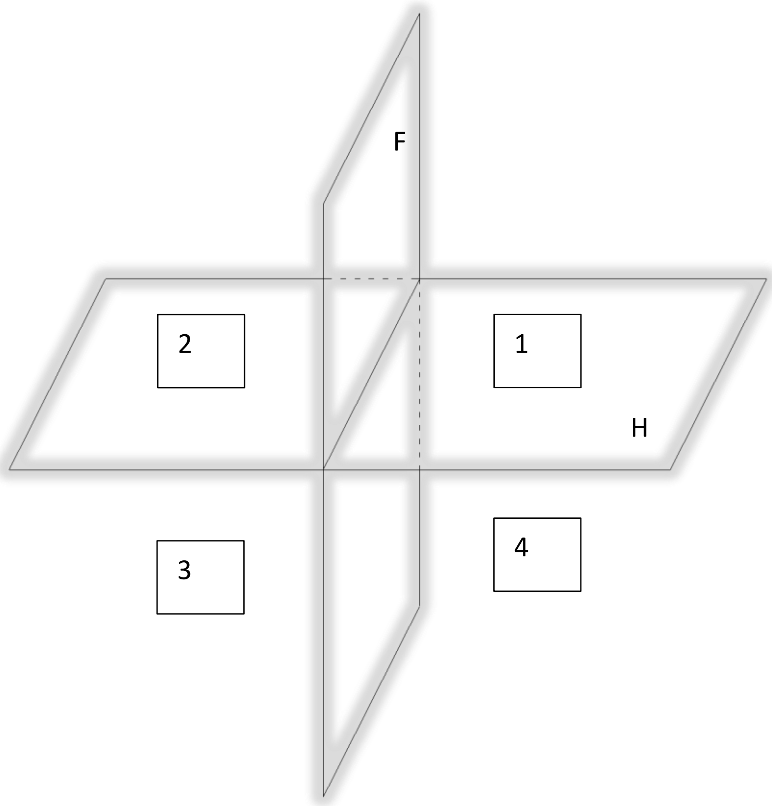


Figure 14 : source author

The two planes (H) and (F) determine four dihedra. Assuming that plane (H) is physically horizontal, we can imagine an observer standing on it in one of the half-planes bounded by the horizon line. We will say that the observer is in the first dihedron.

We will then call:

- The second dihedron, the dihedron at the same distance as the first relative to (H).

- The third dihedron, the dihedron opposite to the first along the edge.
- The fourth dihedron, the dihedron opposite to the second along the edge.

Finding points and displaying the signs of coordinates in a coordinate system are two uses for quadrants. They offer a methodical approach to determining a point's location in relation to the axes and the origin in two dimensions.

3.2 Octant

In the context of three-dimensional coordinate systems, specifically Cartesian coordinates, the term "octant" is used. An octant is one of eight equal sections of space in a Cartesian coordinate system. The positive and negative values of the three coordinate axes—x, y, and z—define these octants.

Athers définitions :

1. Geometric figure consisting of two half-planes with a common edge. (Cordial Dictionary, n.d.)

2. Used as an adjective: In geometry, resulting from the intersection of two planes. Used as a noun: In geometry, a geometric figure consisting of two half-planes with a common edge. In aviation, the angle formed between a horizontal plane and the plane of an aircraft's wings. (Definition of Dihedron - Encyclopædia Universalis, n.d.)

-l’intersection of three planes :

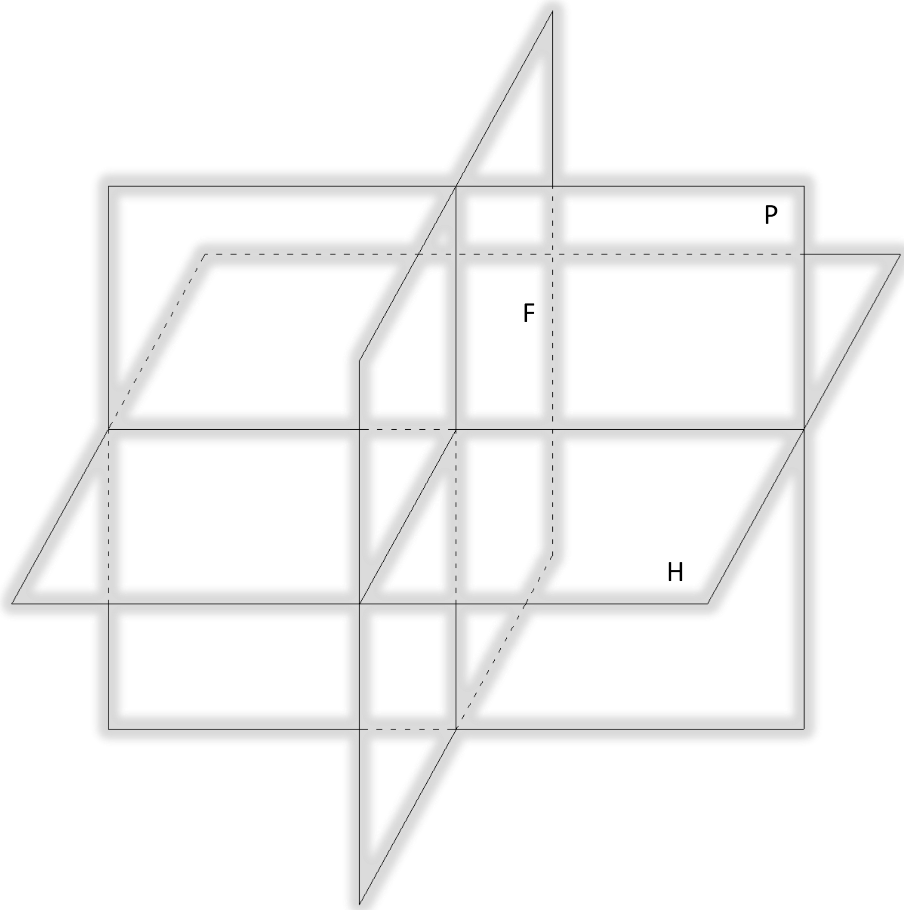


Figure 15 : source author

The positive orientations of the axes are as follows: leftward from point O for the Y-axis, toward the viewer side from the F-plane for the X-axis, and upward from the H-plane for the Z-axis. Conversely, other directions are considered negative.

The projection planes partition the entire space into eight segments known as "octants," with the signs of coordinates for each octant detailed in the accompanying table :

Table 01 : **source author**

	Horizontal	Frontal	Profil
First octant	+	+	+
Second octant	_	+	+
Therid octant	_	_	+
Fourth octant	+	_	+
Fifth octant	+	+	_
Sixth octant	_	+	_
Seventh octant	_	_	_
Eighth octant	+	_	_

To graphically represent an object with the horizontal plane positioned above it or behind the frontal plane, as well as in the case of the profile plane, it is essential to utilize established conventions (refer to the table 1). Once and for all possibilities in space, we specify on which side of the projection plane each considered point is located. Thanks to these conventions, the interpretation of a blueprint becomes unambiguous. The blueprint is positioned to have positive and negative directions for the three axes X, Y, and Z.

<u>EXERCICE :</u>

a/ Project the following points :

A (4,3,2) ; B (3,-3,5) ; C (2,4,-6) ; D (5,-2,-4) ; E (7,0,-3) ; F (6,2,0) ; G (9,-3,0) ; H (10,0,4) ; L (11,5,0) ; M (1,3,-3) ; N (8,-4,4).

b/ determine the horizontal projection, frontal projection and the profil projection :

A (5,1,2) ; B (3,3,-6) ; C (-2,4,6) ; D (-4,2,-1) ; E (8,10,-3) ; F (0,2,0) ; G (0,-3,0) ; H (1,0,-4) ; L (-3,-5,0) ; M (-1,8,-3) ; N (6,-5,-4).

c/ Determine the coordinates of each of the following points. Draw the three projections in the following cases:

1. A point to the right of the profile projection plane, 2 cm from it, in front of the frontal plane, 5 cm from it, and below the horizontal plane, 3 cm from it.
2. A point to the right of the profile projection plane, 6 cm from it, behind the frontal plane, 3 cm from it, and in the horizontal plane.
3. A point on the Y-axis and to the left of the profile plane, 2 cm from it.
4. A point on the Z-axis and below the horizontal plane, 2 cm from it.

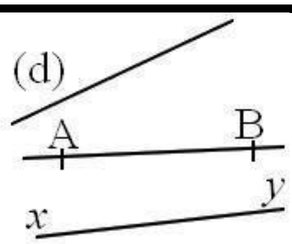
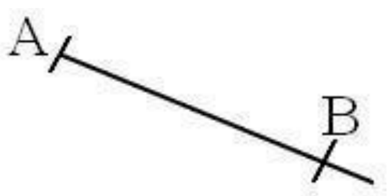
COURSE N°03 : THE LINE AND NOTABLE LINES

1 LINE :

Definition:

Line is a set of aligned points; knowing the position of two points belonging to this line is sufficient to draw its spatial configuration. Thus, the line is an unlimited geometric object in space. In the table below, we can see several other planimetric objects that can define a line.

Table 2: source author

Objet	Representation	rating	discription
Point		A	The smallest body in descriptive geometry; point is the exact location where the two lines of the cross intersect.
line		(d) (AB) (xy)	**Line (d) (AB) (xy)** The line (AB) is the straight line that passes through points A and B. A line is unlimited on both sides.
Half-droite		[AB)	**Half-Line [AB)** The half- line [AB) is a portion of the line (AB) bounded by point A. A is called the origin of the half-line.

2 Line projecting on the three projection planes :

A line in space also is formed by the intersection of two planes, and it is not constrained or limited.

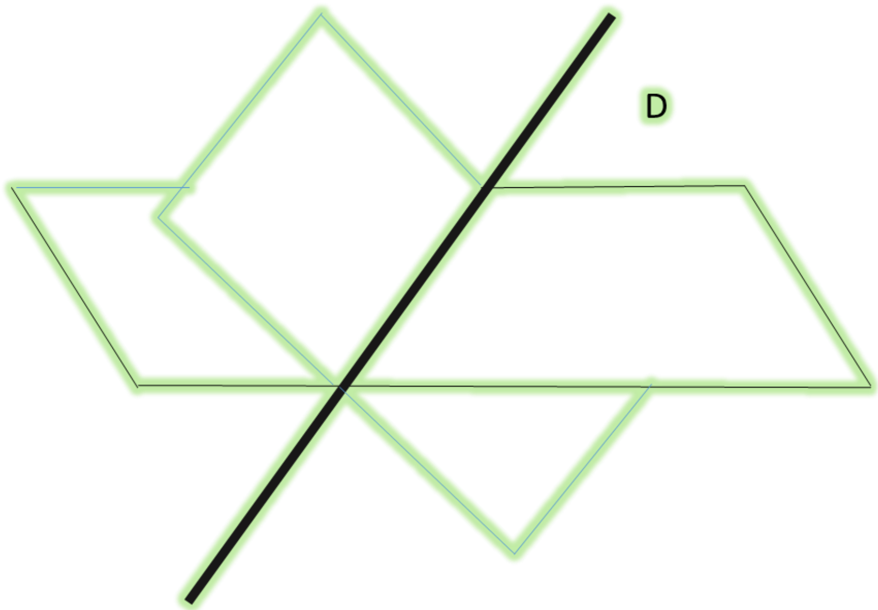


Figure 16 : **source author**

- ***A spatial projection of any oblique line: D***

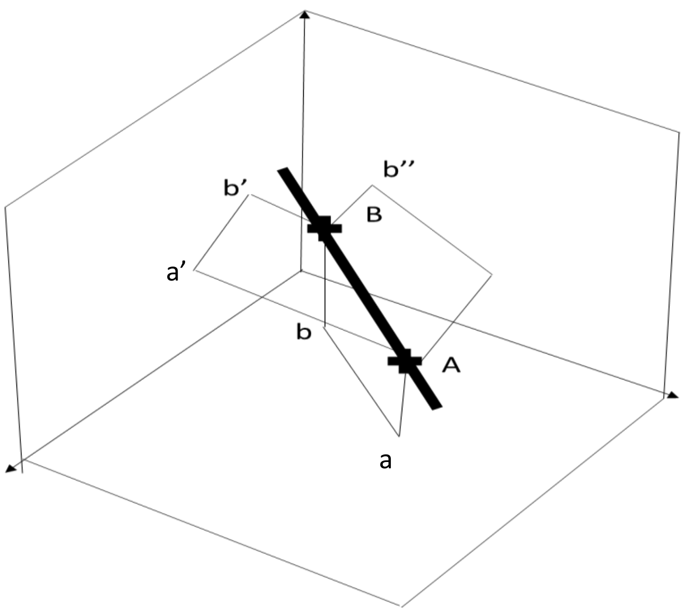


Figure 18: **source author**

ab, a’b’, a’’b’’ are the projections (figure18) of this line on the horizontal, frontal plane and that of the profile

In general, the common rules of lines projection use just two points for drawing any line projections because as line is a succession of points, its projection on any plane will be determined by projection each point of the line on that plane.

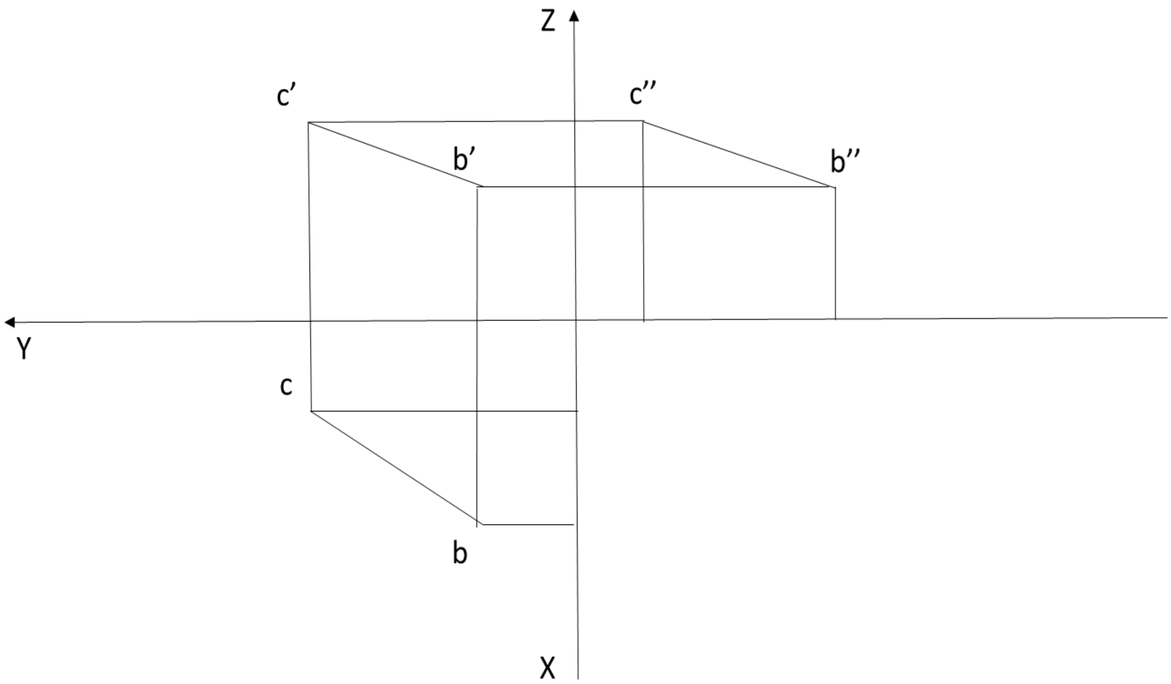


Figure 19 : **source author**

3 Remarkable Lines:

In descriptive geometry, a line parallel to the horizontal plane is referred to as horizontal, while a line parallel to the frontal plane is called frontal. Additionally, a line parallel to the profile plane is termed as the profile line. Similarly, a line perpendicular to the horizontal plane is named vertical, and a line perpendicular to the frontal plane of the projection is labeled as upright.

❖ THE VERTICAL LINE

d', d" ⊥ y→ The true length of a segment belonging to this line will be measured on its projections d' and d".

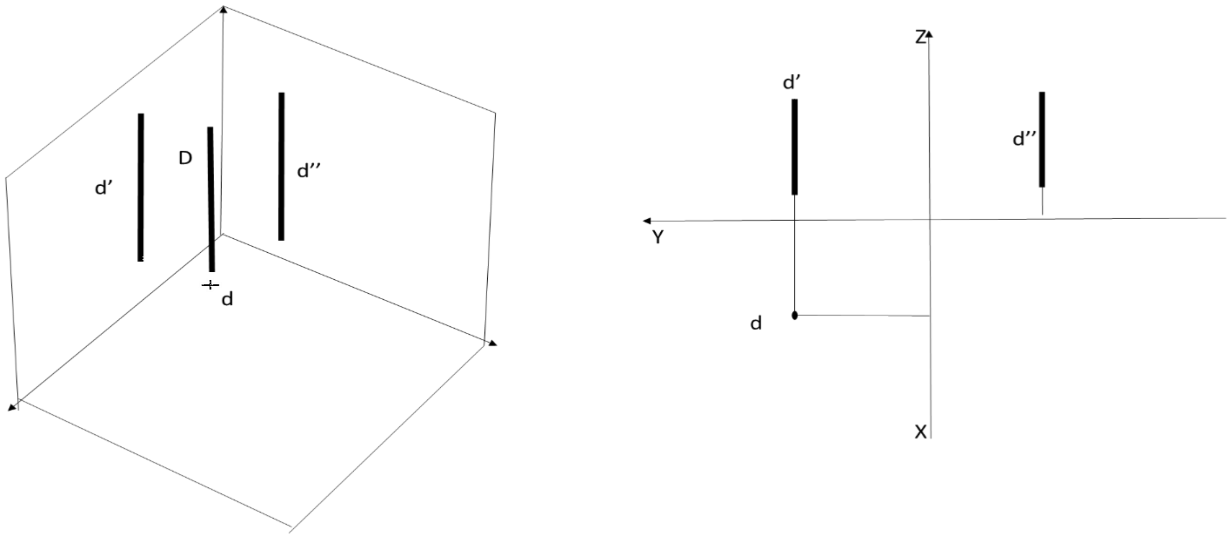


Figure 20 : source author

❖ THE UPRIGHT(standing) LINE :

d ⊥ y→ and d" // y→ The true length of a segment belonging to this line will be measured on its projections d and d".

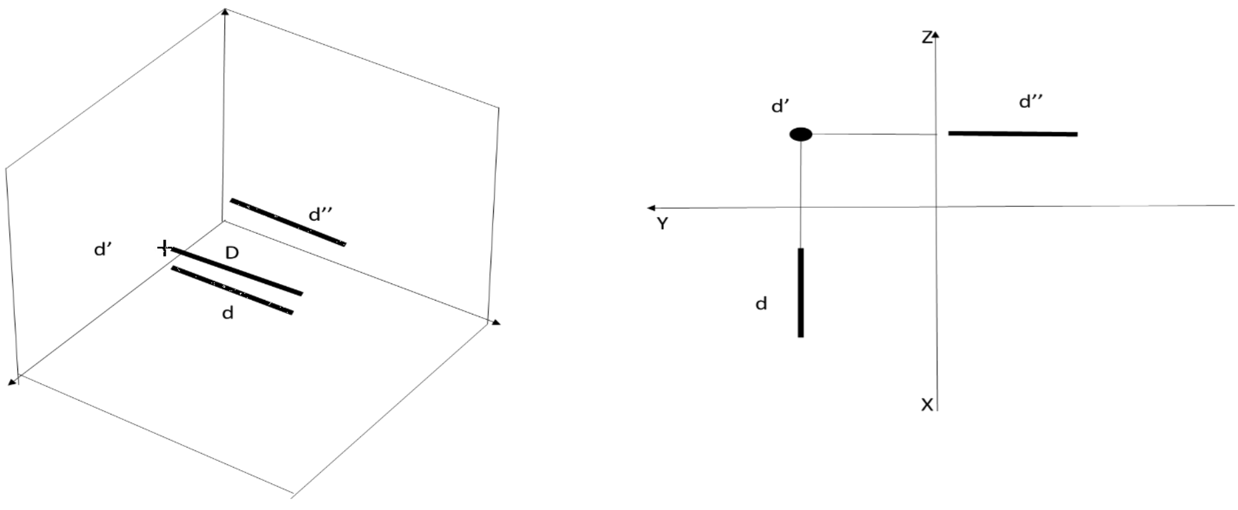


Figure 21 : source author

❖ THE HORIZONTAL LINE

d’, d" // y → and the projection d forms two angles of inclination s° and p°, where s° is the angle of the horizontal line with the profile, and p° is the angle of the line with the frontal plane. The true length of a segment belonging to this line will be measured on its projection d.

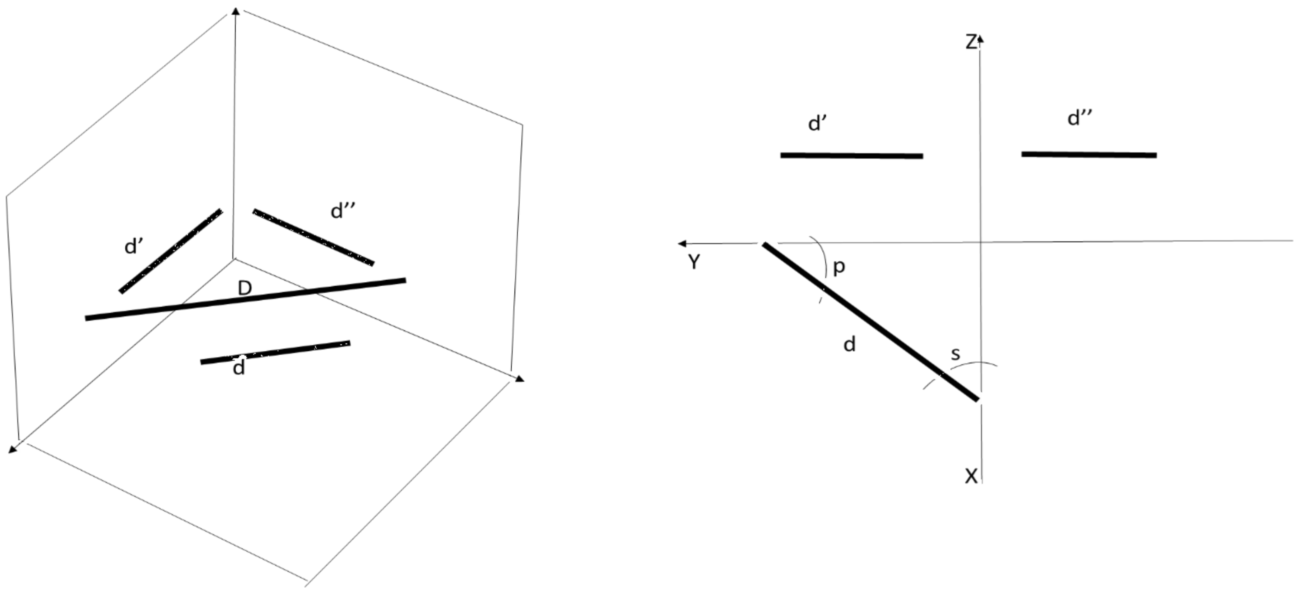


Figure 22 : source author

❖ FRONTAL LINE :

d // y , d" ⊥ y → and the projection d’ forms two angles of inclination s° and p°, where s° is the angle of the frontal line with the profile, and p° is the angle of the line with the horizontal plane. The true length of a segment belonging to this line will be measured on its projection d’.

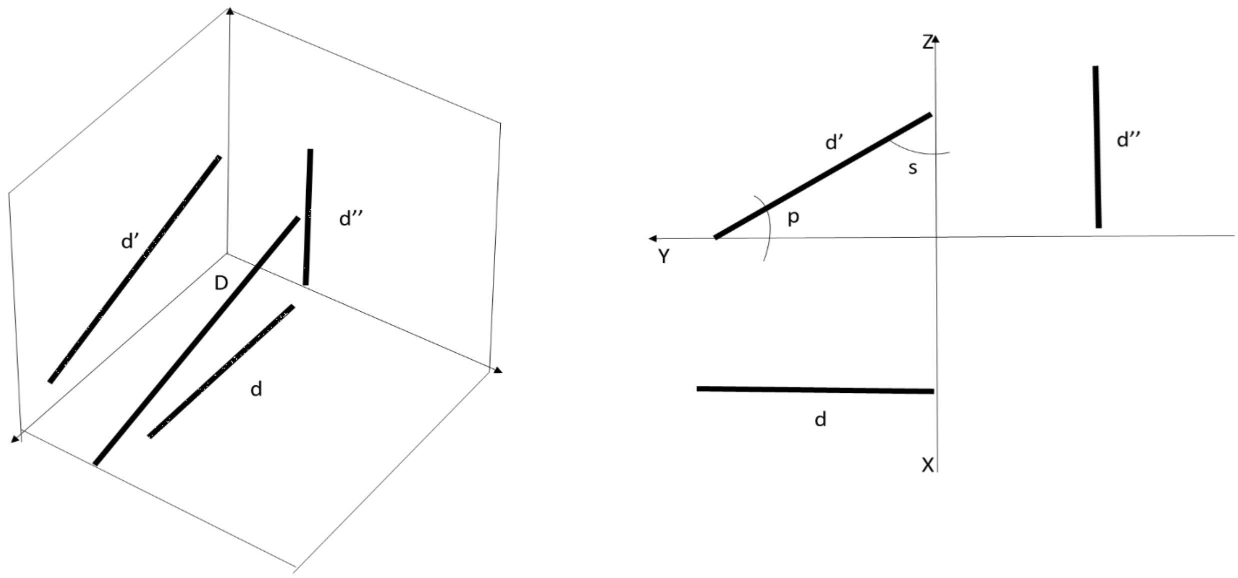


Figure 23 : source author

❖ PROFIL LINE :

d ⊥ y →, d’ ⊥ y →, and the projection d’’ forms two angles of inclination p° and s°, where s° is the angle of the profile line with the frontal, and p° is the angle of the line with the horizontal plane. The true length of a segment belonging to this line will be measured on its projection d’’.

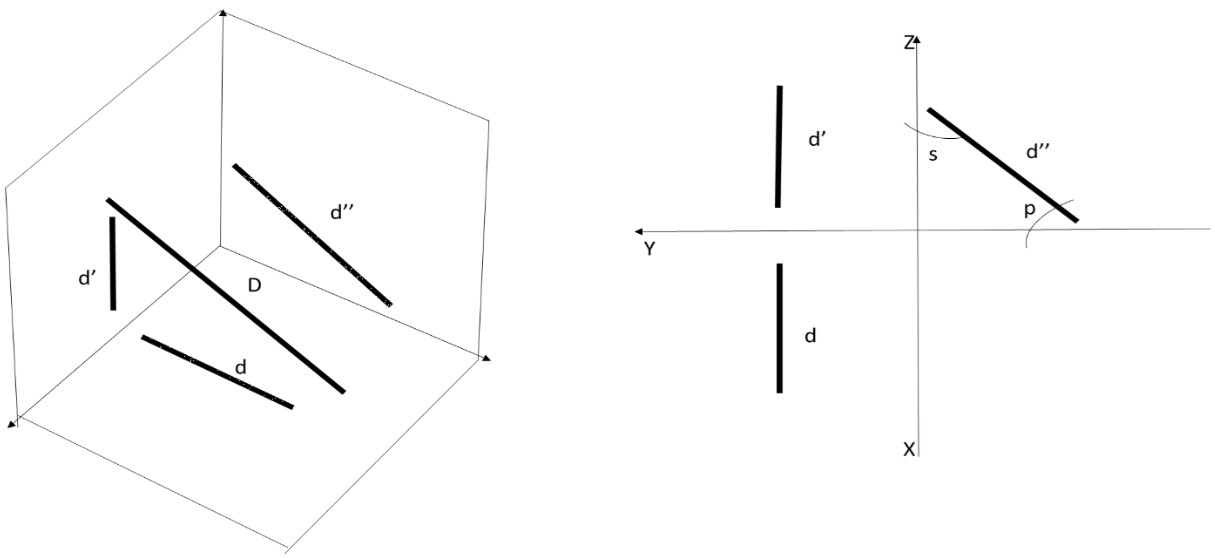


Figure 24 : source author

❖ PARALLEL LINE to two planes of projection:

d // $\vec{y}$, d' // $\vec{y}$

The true length of a segment belonging to this line will be measured on both its projections d and d', as it is a line that is both horizontal and frontal.

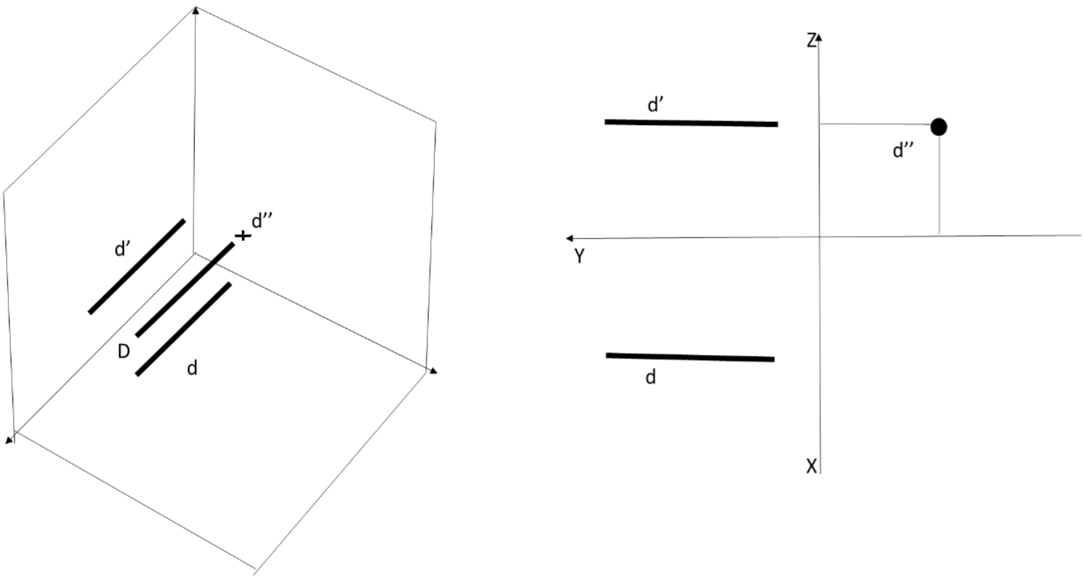


Figure 25: source author

Notes takes :

For a line D to pass through a point M, its horizontal projection must intersect the horizontal projection of point M. This condition also holds true for the frontal and profile projections. To establish the determination of an unknown line, it is essential that either the horizontal and frontal projections, frontal and profile projections, or horizontal and profile projections align for two points or two projections of the line.

COURSE N°04 : THE PIERCING POINTS OF A LINE

1 PIERCING POINTS :

Ageneral line in space intersects the projection planes in the specific points that are called « trace » or « peircing-point ».

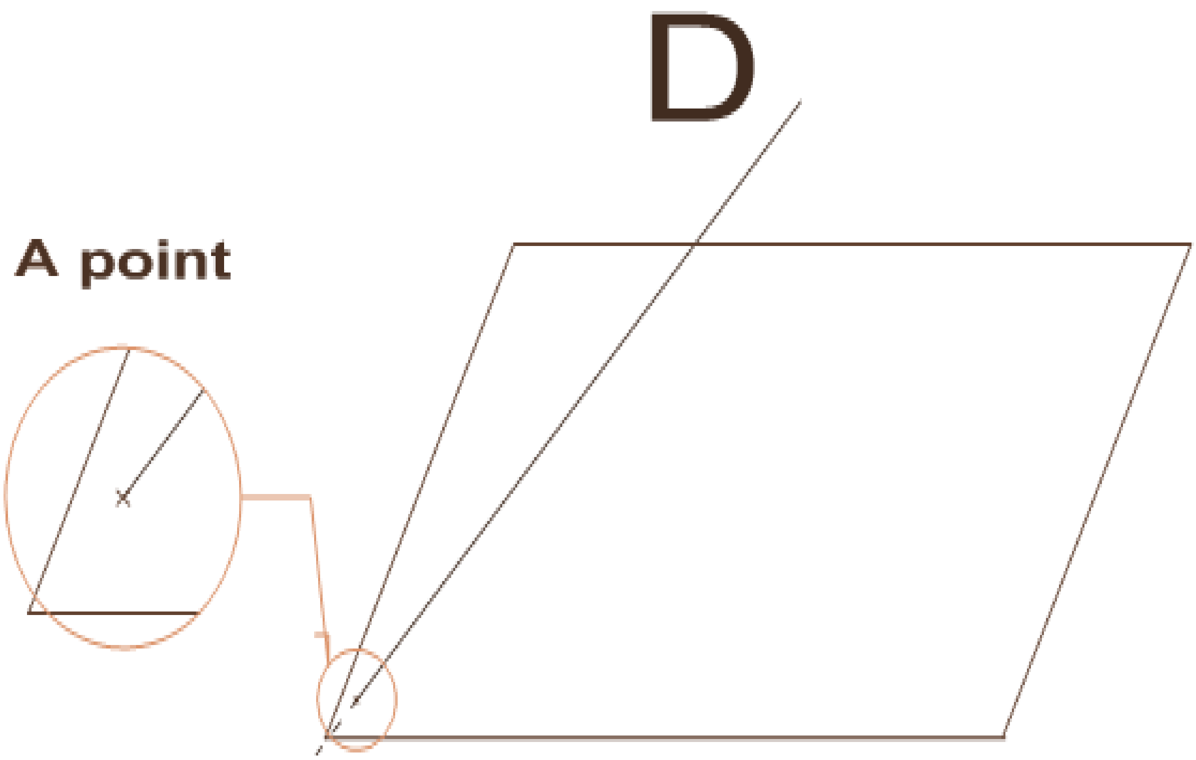


Figure 26: source author

2 Features of Points of Piercing:

Coordinates: The coordinates of each pierced point on the coordinate planeindicate its location with respect to the x, y and z axes.

Piercing points show the intersection of two geometric entities. If it's lines or plane, **the intersection of the lines is always a piercing point.**

Multiplicity: A line may occasionally cross another line or plane more than once. The type of geometric entities involved determines how many piercing points are necessary.

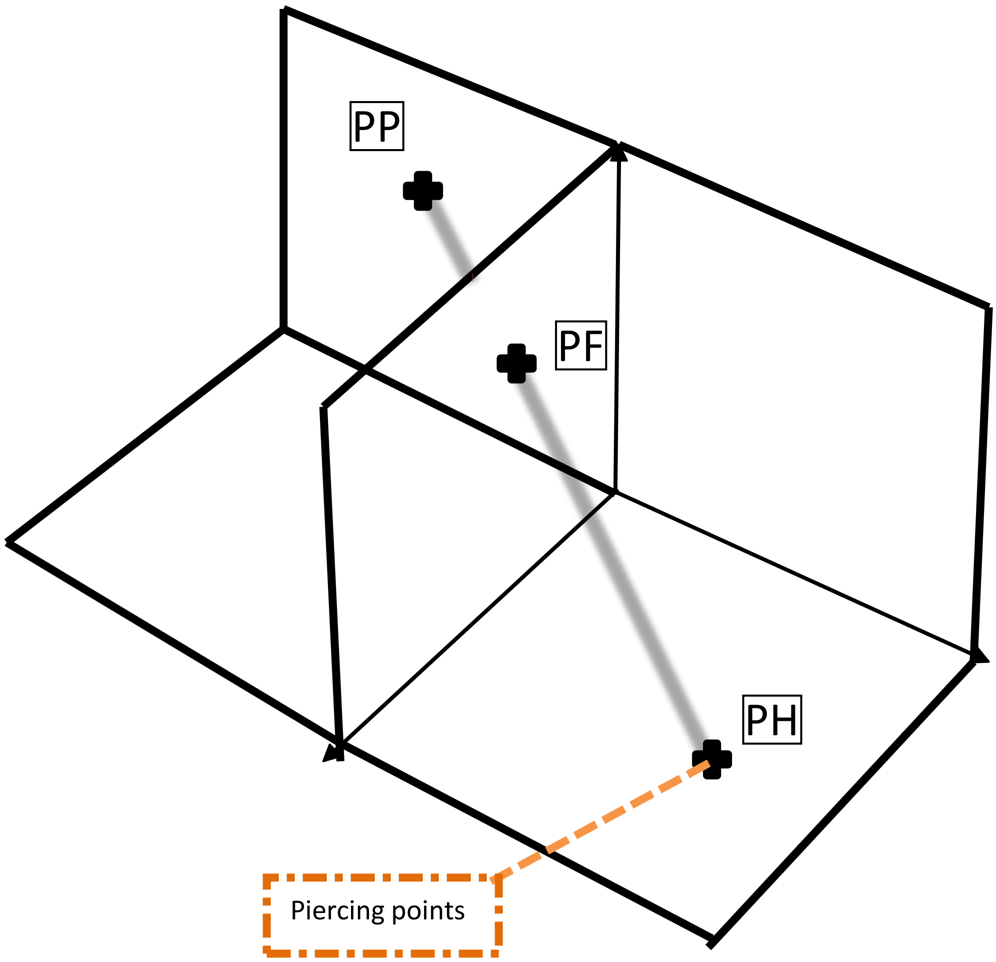


Figure 27: source author

3 Horizontal and Frontal Traces in the Case of Two Projection Planes

Suppose line D projects onto the horizontal plane as line "d" and onto the frontalplane as line "d'." To find the horizontal and frontal traces, follow these steps:

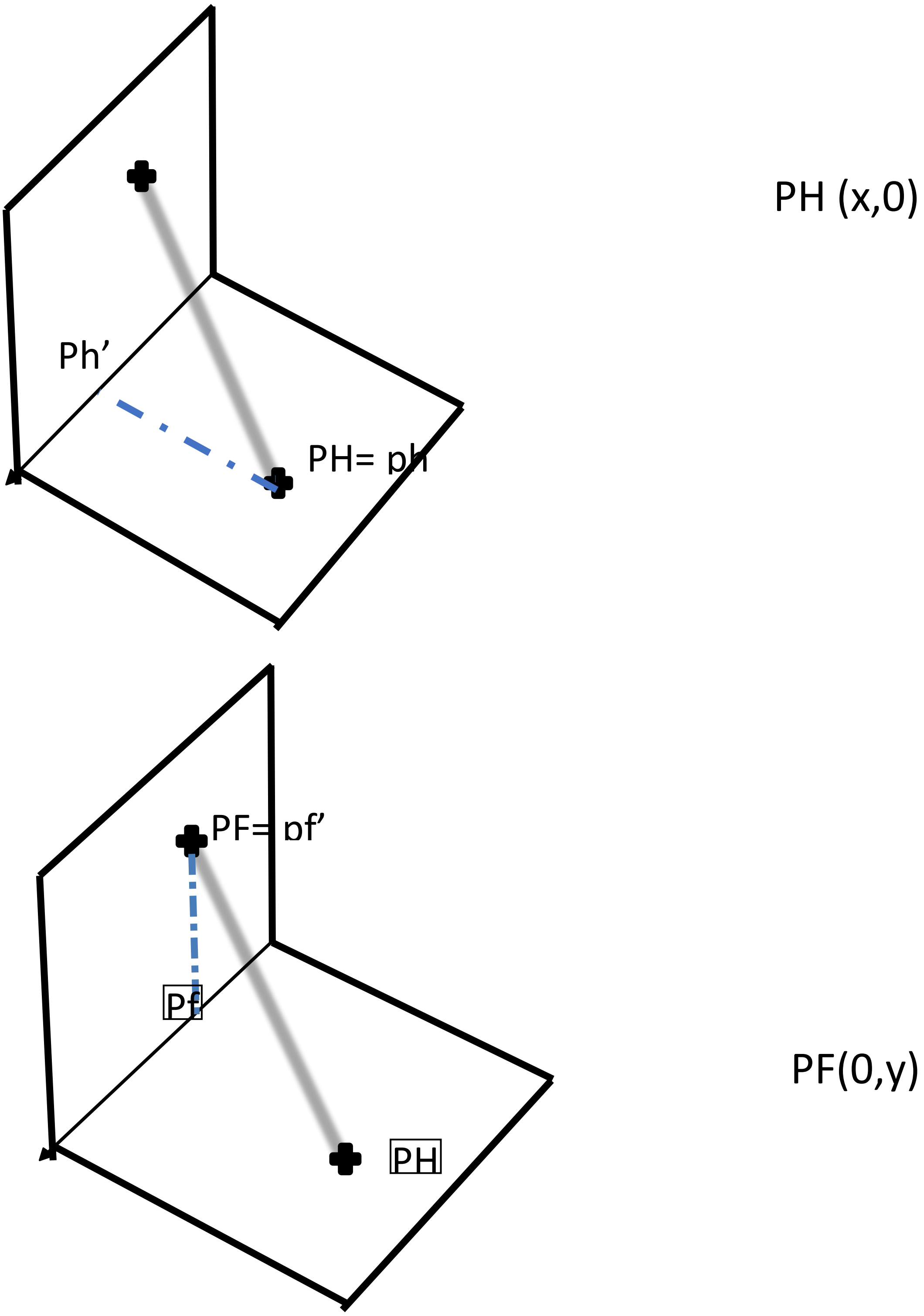


Figure 28 : source author

- Extend the projection "d'" along the y-axis. The intersection point is th', in this case, theline is at a greater distance from the origin (i.e., an elevation in this case) = (x,0).
- Next, make parallel projection lines that intersect with projection d. This intersectionpoint is th.
- Using the same process, find the point PF (Pf, Pf'). Simply extend the projection d along the y-axis. The intersection point is tf, in this case, the line is at a greater abscissa = (0, y).
- Again, create parallel projection lines that intersect with projection d. This intersectionpoint is Pf'.

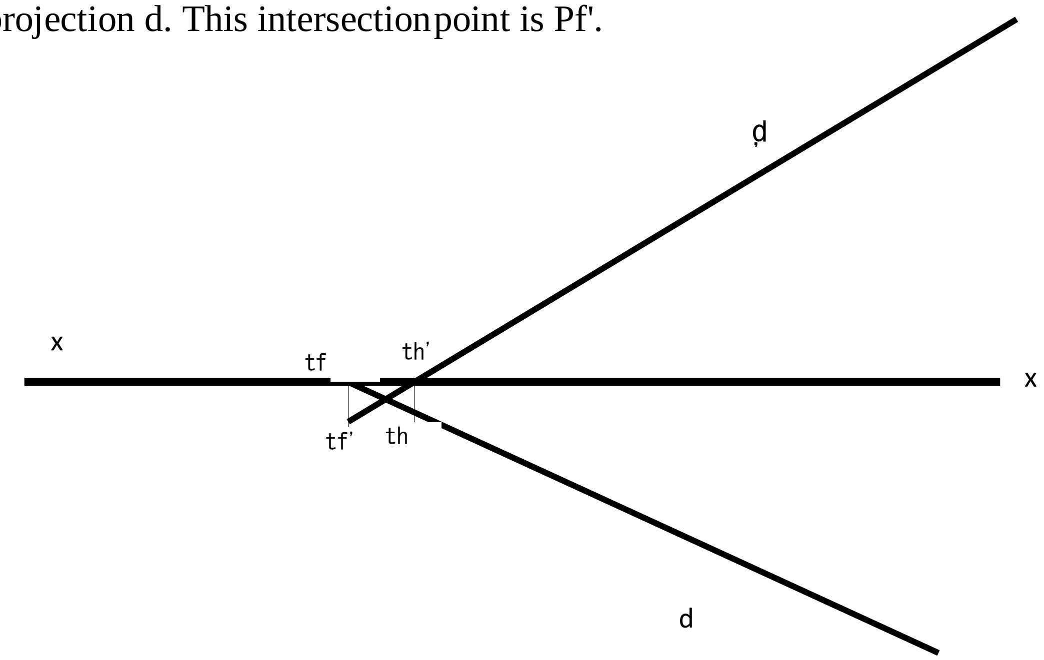
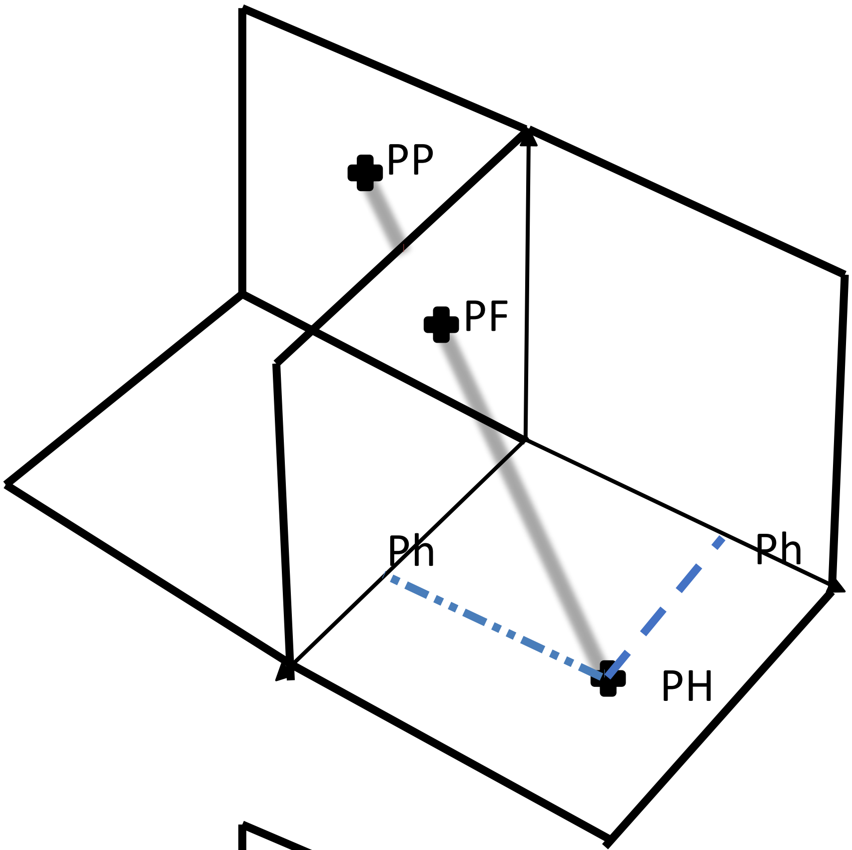
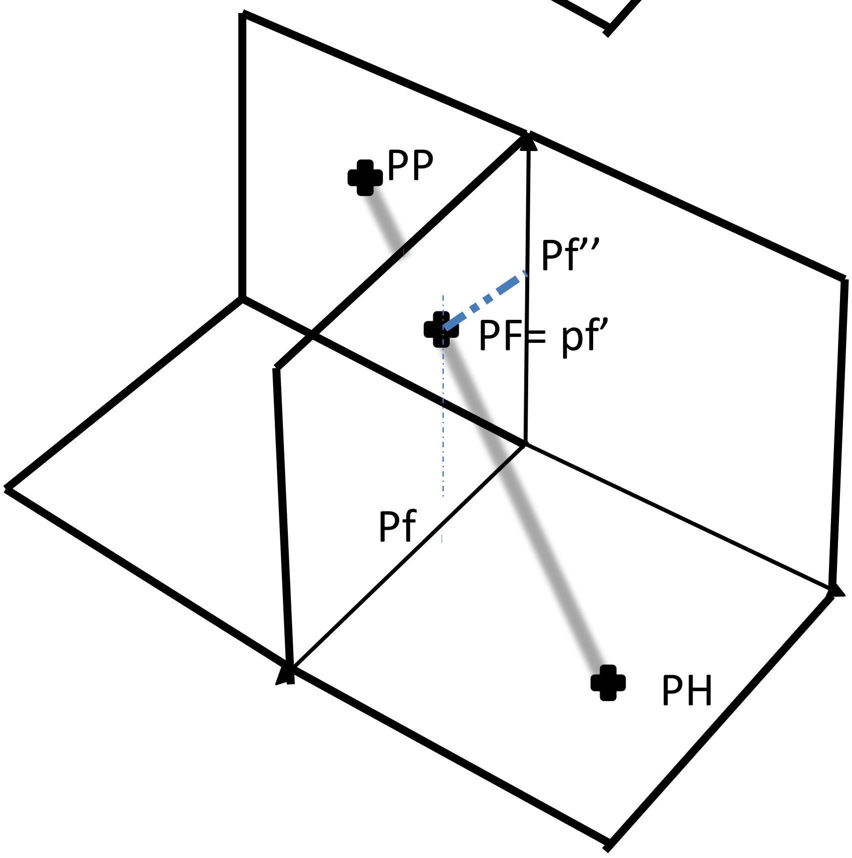


Figure 29: source author

4 Horizontal, Frontal, and Profile Traces in the Case of Three Projection Planes:



PH (x,y,0)



PF (0,y,Z)

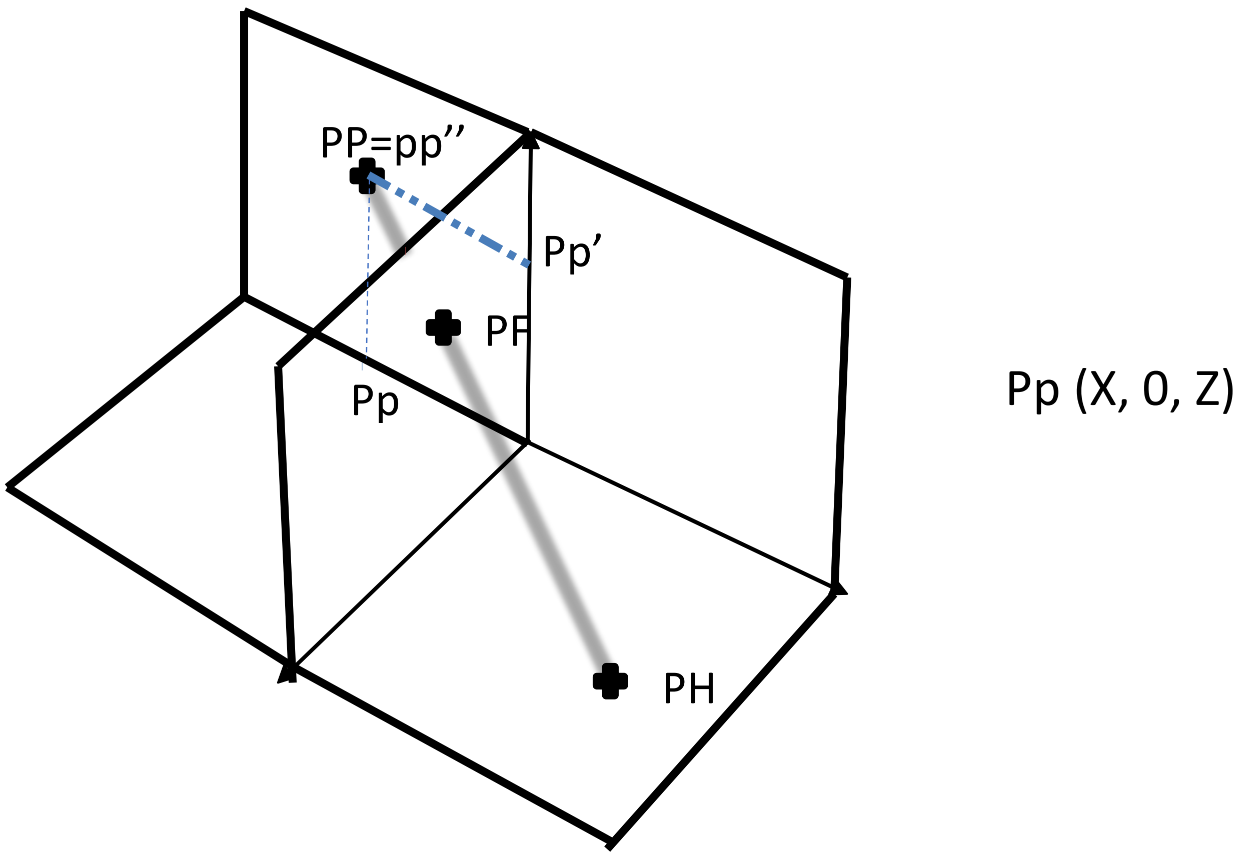


Figure 30: source author

Suppose line D projects onto the horizontal plane as line "d" and onto the frontal plane as line "d'." To find the horizontal, frontal, and profile traces, follow these steps:

- Extend the projection "d'" along the y-axis. The intersection point is th', in this case, the line is at a greater elevation (which means an elevation or level in this case) = (x, y, 0).
- Next, create parallel projection lines that intersect with projection d. This intersection point is th, and th" requires folding along the y-axis.
- Using the same process, find the point TF (tf, tf', tf"). Simply extend the projection d along the y-axis. The intersection point is tf, in this case, the linehas a greater abscissa = (0, y, z).
- Again, create parallel projection lines that intersect with projection d. This intersection point is tf'.

- Use the same process to find the point TP (tp, tp', tp"). Simply extend theprojection d along the x-axis. The intersection point is tp.

tp requires folding along the y-axis, similarly, extend d along the z-axis. This intersection results in the point tp'. In this case, the line is at a greater distance = (x, 0, z).

- Drawing guide lines allows us to find an intersection, which is tp" .

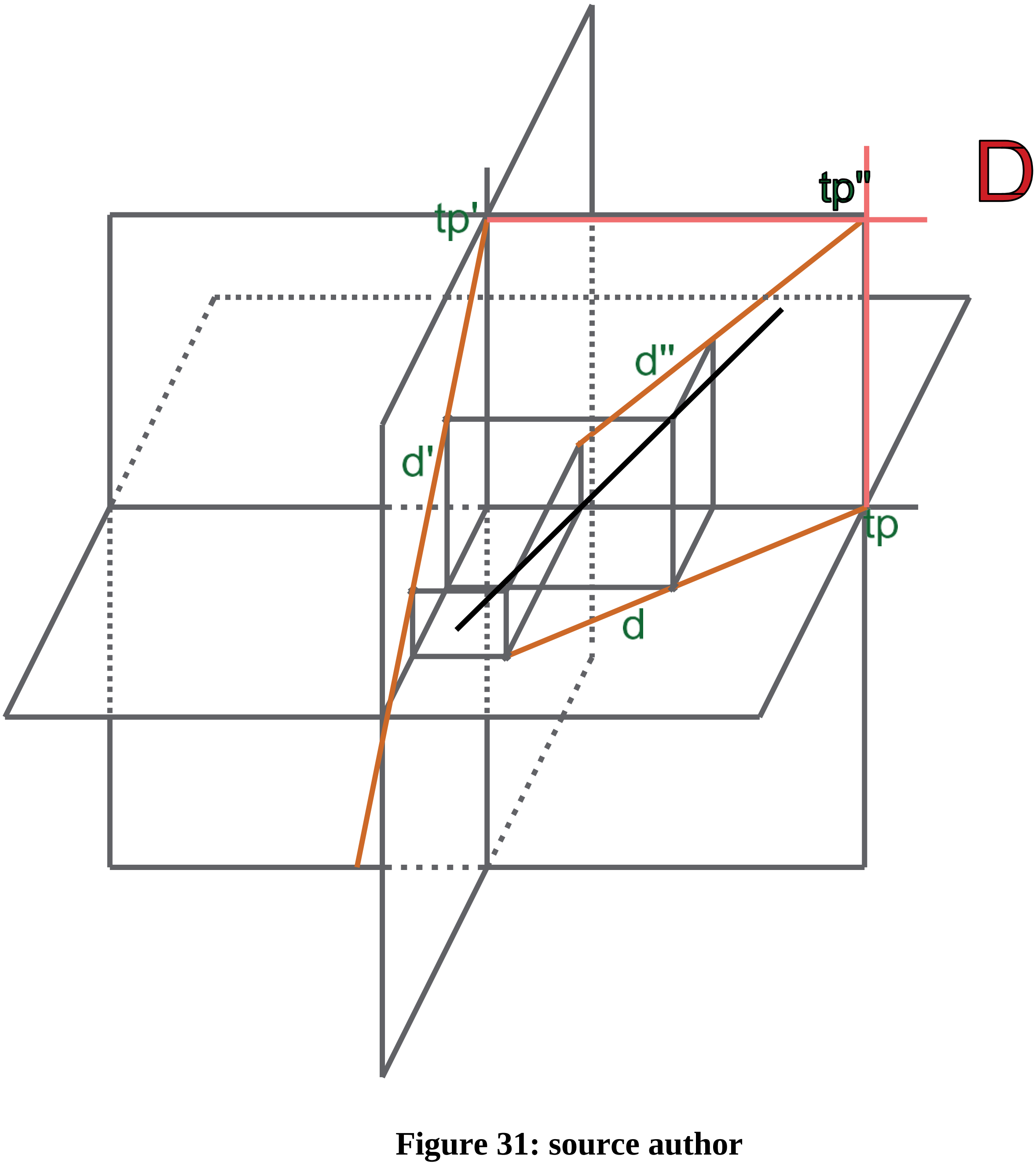


Figure 31: source author

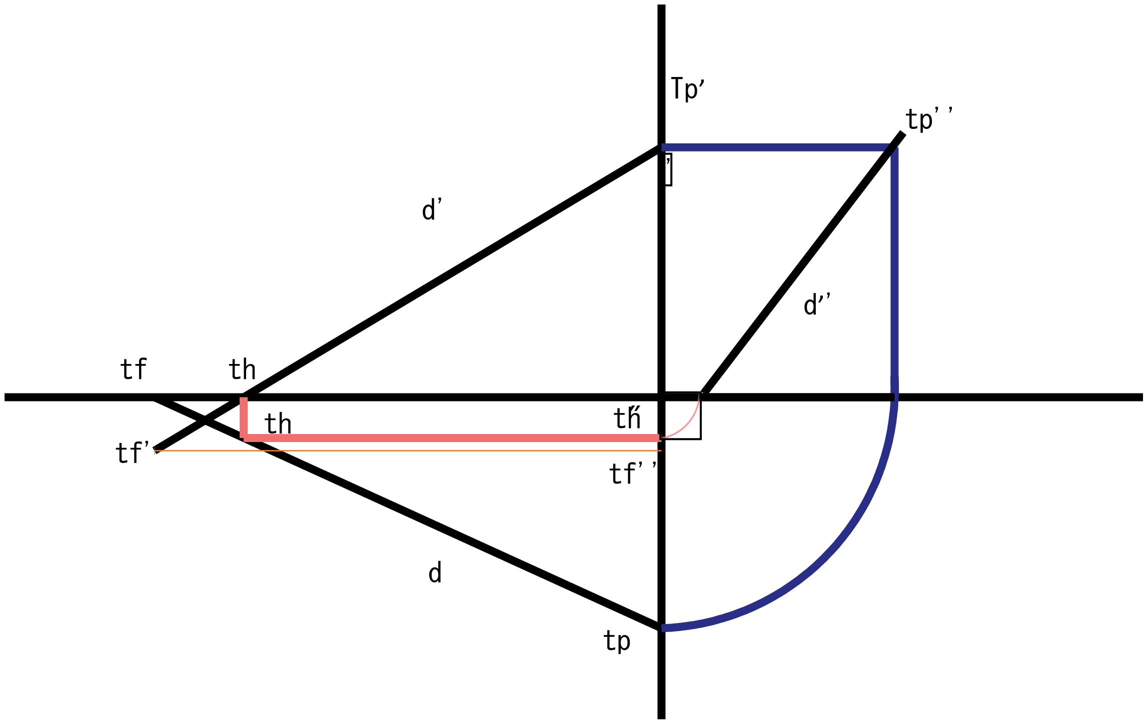
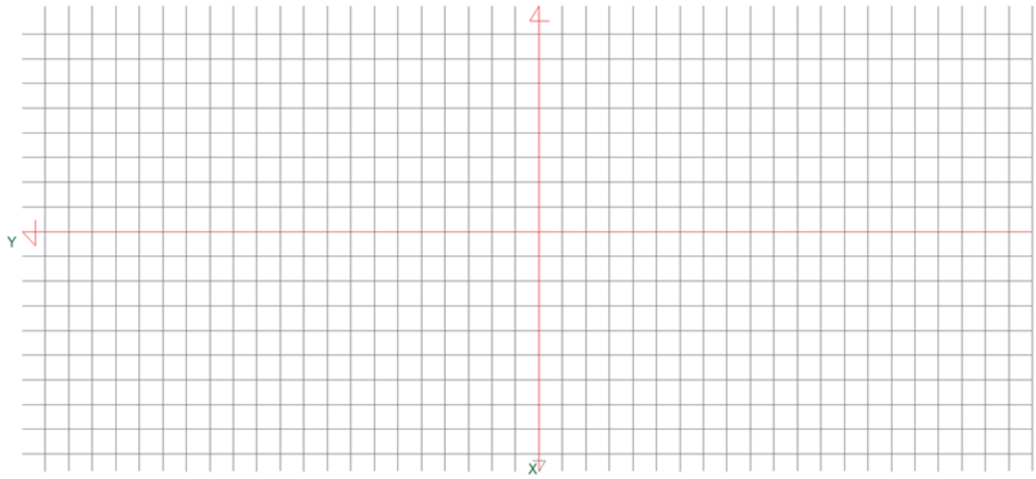


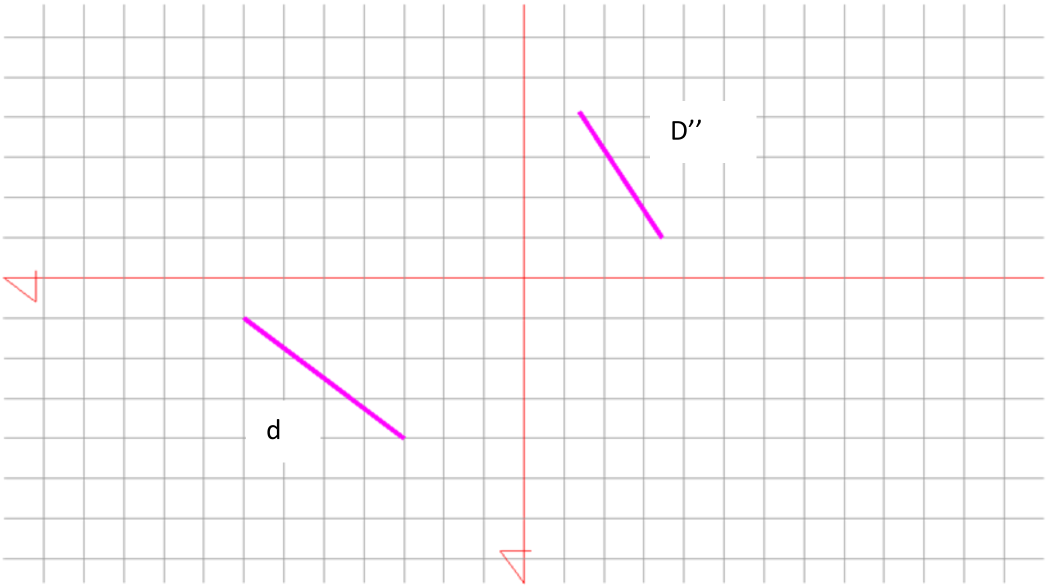
Figure 32: source author

Exercice:

Given two points Tp (-2.0.-3) and Tf (0. +2. +3), find on the drawing the projection of a line S (s. s'. s ") defined by Tp and Tf?"



- To identify on the provided figure the outlines of line D and its third projection.



1 Position of line in space:

Two lines in space can be in one of two general positions: a) They can lie within the same plane and either intersect or be parallel to each other. b) They can be positioned in a way that no plane can pass through both lines, meaning they neither intersect or they are parallel to each other.

.1.1 Parallel lines:

Two lines in space are parallel if their projections onto a plane are parallel due to the parallelism of their corresponding planes. Similarly, if the horizontal or frontal projections of lines onto the same plane are parallel, then the lines in space are also parallel.

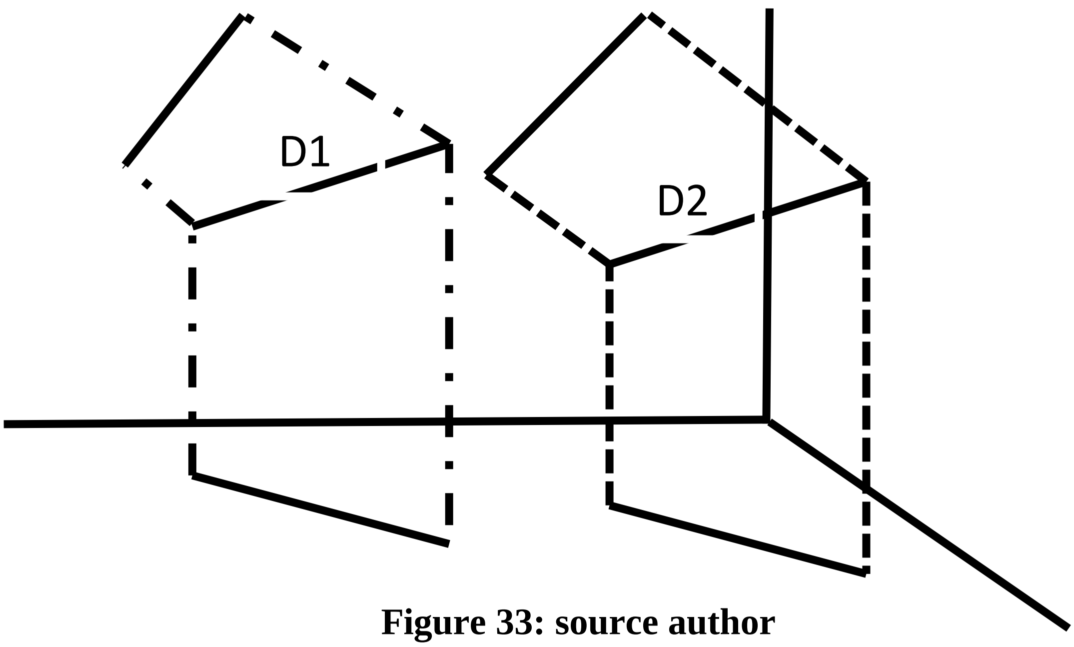


Figure 33: source author

For example:

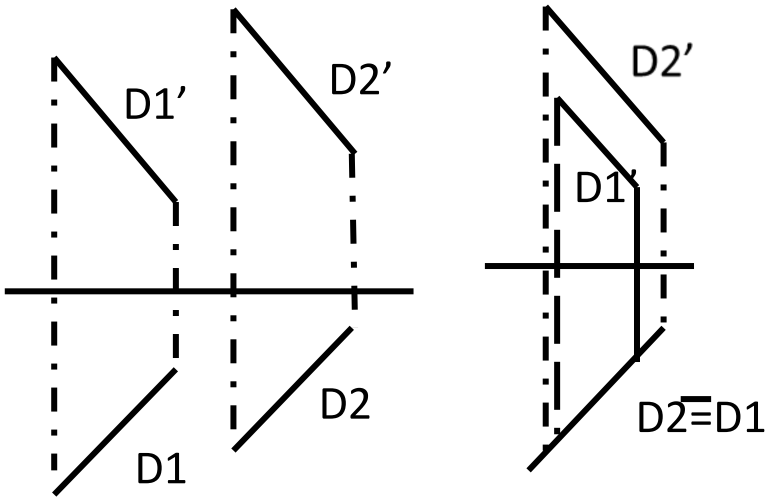


Figure 34: source author

The paralle position for profile lines (remarkable lines) is defined on the profile projection

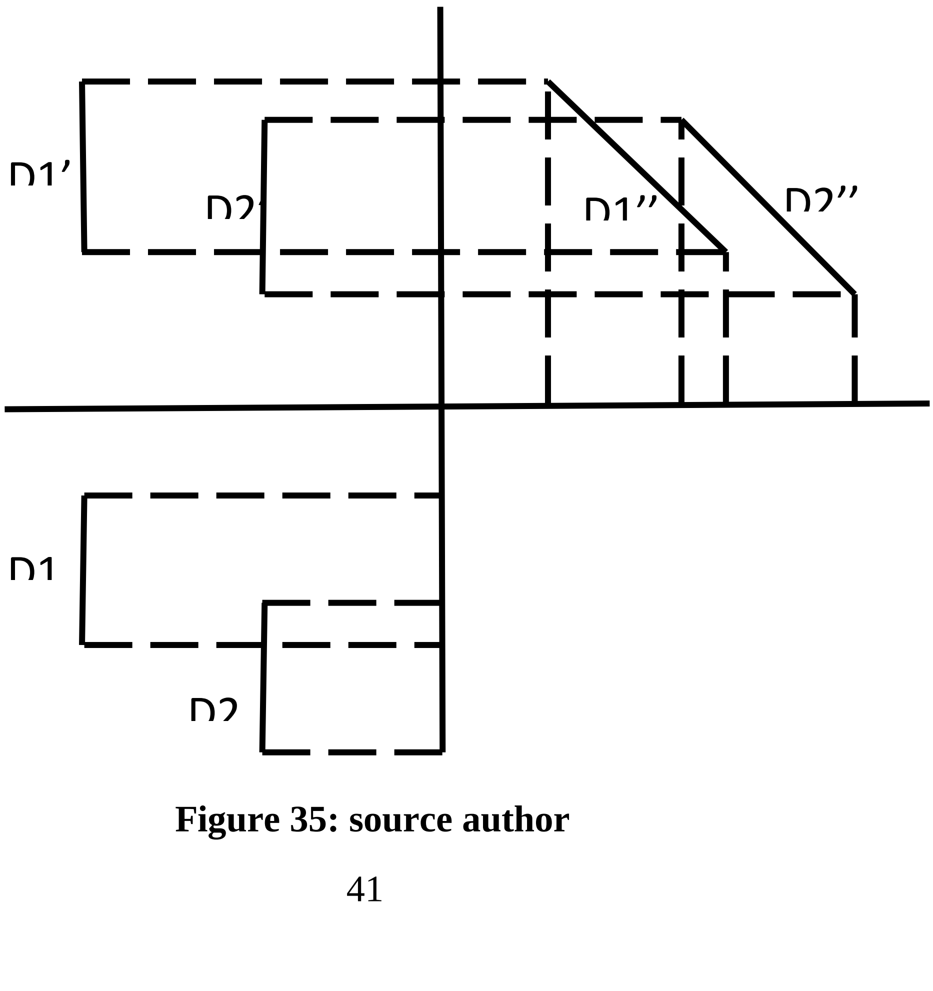


Figure 35: source author

.1.2 Intersect lines:

Two lines intersecting in space create projections that intersect at points lying along the same line perpendicular to the ground.

Points of intersection:

Points of intersection are important in geometry for a number of reasons.

Geometric constructs: In geometric constructs, points of intersection are essential. For example, constructing a perpendicular bisector entails drawing a line perpendicular to the segment and locating the midway, or intersection point.

Points of intersection are useful for understanding and analyzing geometric shapes in figures and shapes analysis. For example, the point where a line and a circle cross can reveal details about their respective positions.

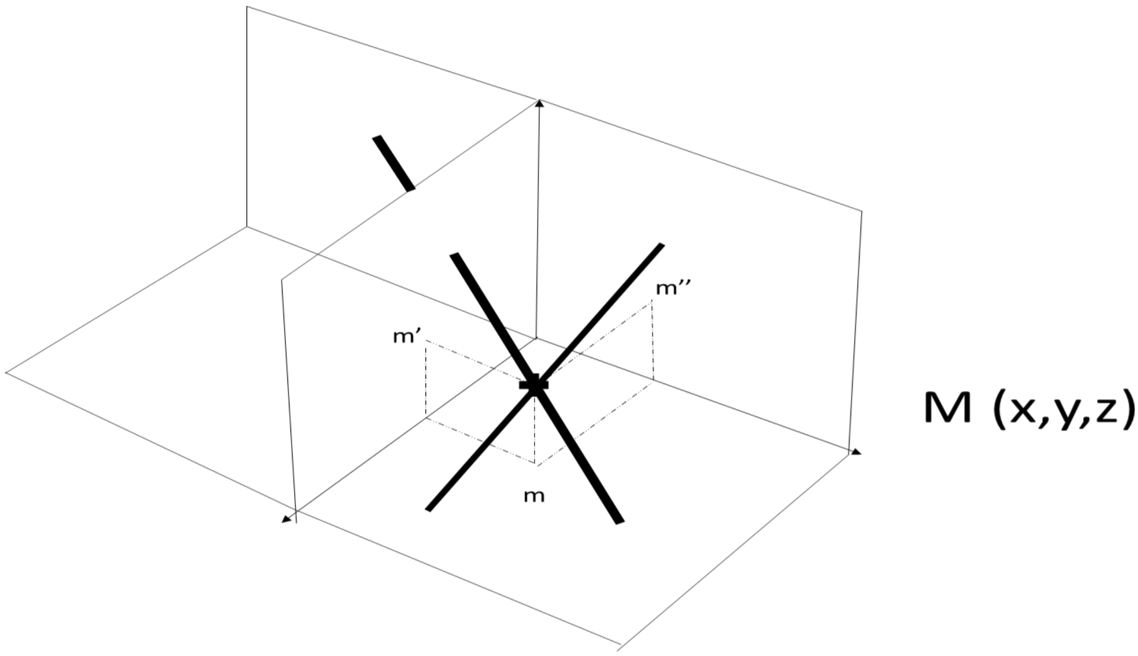


Figure 36: source author

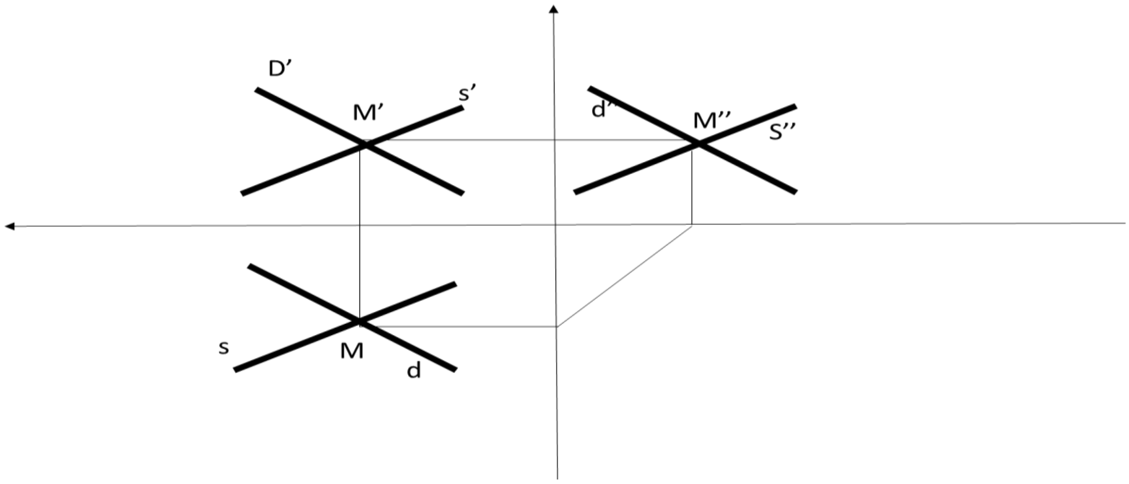


Figure 37: source author

.1.3 Skew lines:

are defined as two lines in space that neither intersect nor are parallel. W hen considering their projections, the point where the projected lines intersect does not fall along the perpendicular common to the ground lines.

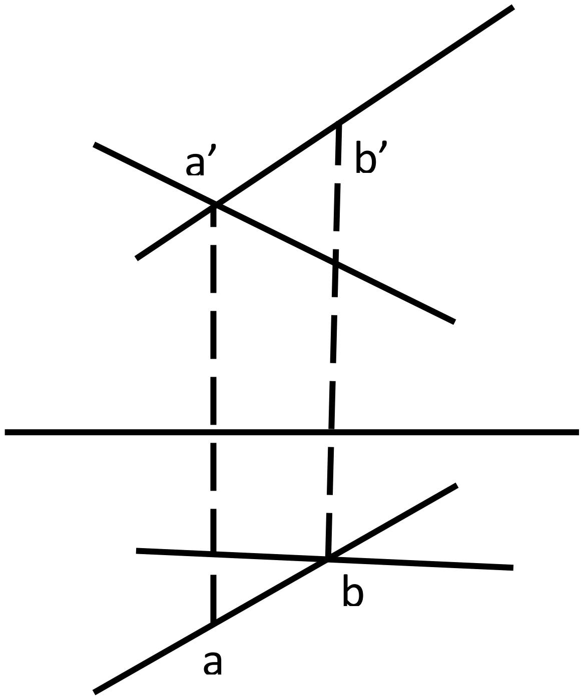


Figure 38: source author

EXERCICE :

- what are the main propreties of line in space ? describe the piercing point ?
- draw three position of lines in the space ?
- draw an horizontal , frontal, and profile lines ?
- which lines in space can be named a skew lines ?

COURSE N°06: A plane and remarkable planes

1 Determination of plane:

The location of a plane in space can be determined by using various techniques :

- tree points

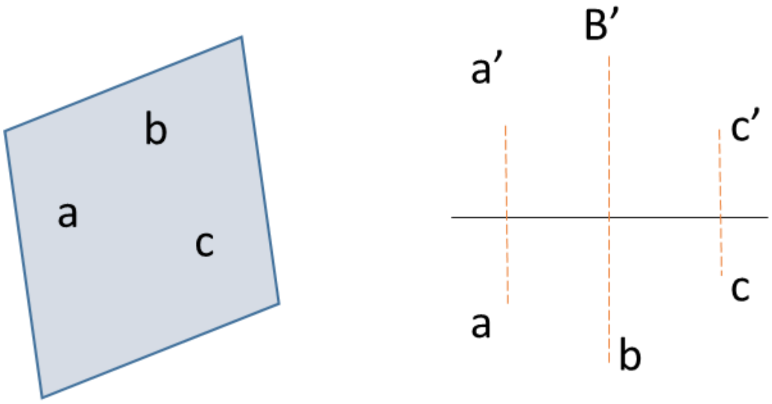


Figure 39: source author

- line and a point

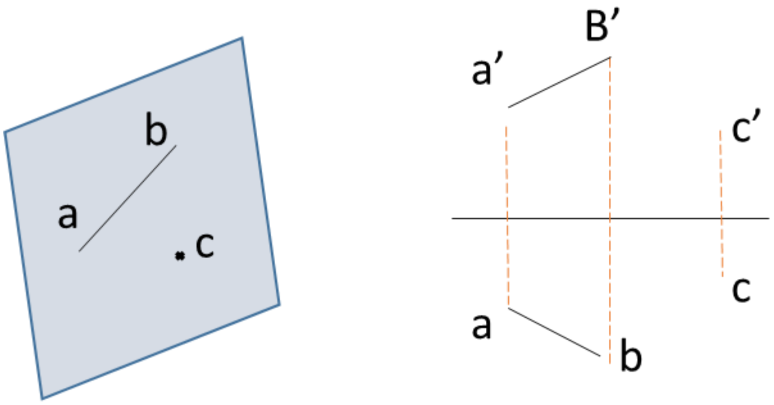


Figure 40: source author

- two intersecting lines

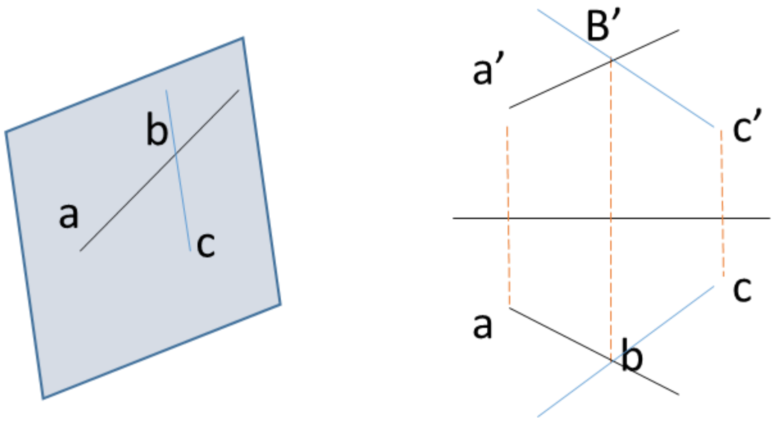


Figure 41: source author

- two parallel lines

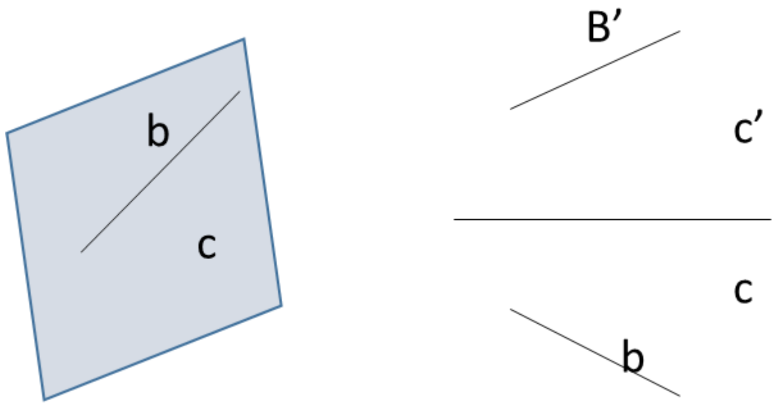


Figure 42: source author

- planar figure

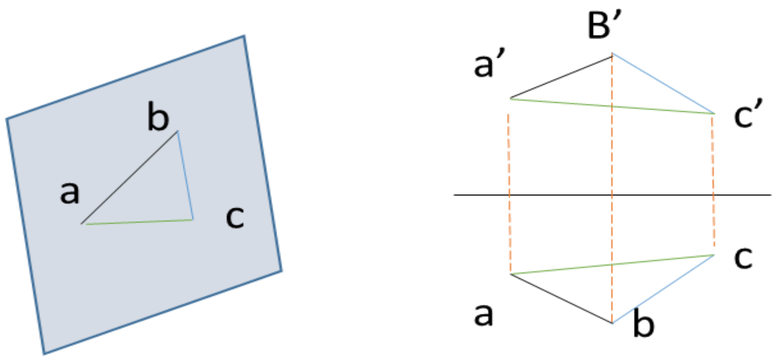


Figure 43: source author

- plane traces

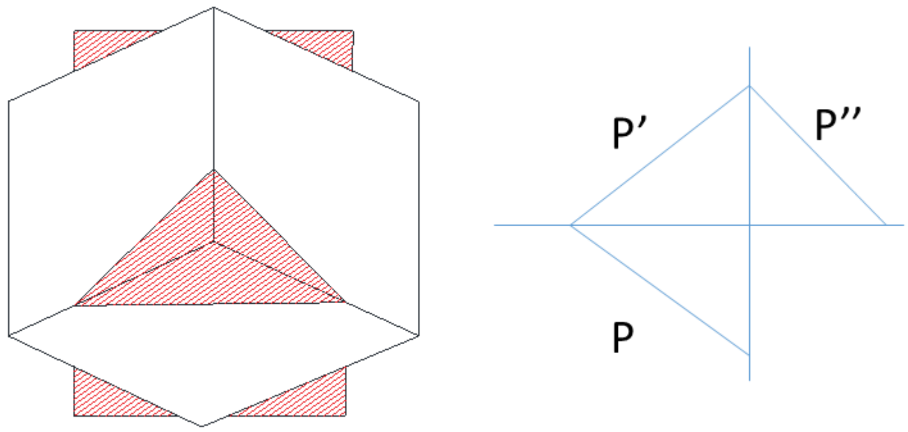


Figure 44: source author

.1.1 Plane traces:

The lines where a plane intersects the projection planes are known as plane traces (figure 44), commonly used to position the plane. Typically, planes have three traces: horizontal, frontal, and profile. It's important to note that : the traces of a line within a plane align with the related plane traces. These traces can be determined by projecting lines associated with the plane. The angle of inclination is not necessarily indicated by the traces. Plane traces are useful for determining a plane's position except when they coincide with ground lines, as they project two straight lines from the plane, which may intersect or be parallel to each other.

two intersecting lines (figure 45). We obtain the traces of the plane defined by these lines by seeking the horizontal and frontal traces of these lines. The horizontal traces ph1 and ph2 of the lines belong to the horizontal plane of projection and the plane defined by these lines. Therefore, they belong to the horizontal trace of this plane. The horizontal trace of the plane is thus the line joining the two horizontal traces of the lines (figure 46). Similarly, the frontal traces pf'1 and pf'2 of the lines belong to the frontal plane of projection and the plane defined by these lines. Therefore, they belong to the frontal trace of this plane. The frontal trace of the plane is thus the line joining the two frontal traces of the lines.

When the traces of a plane are not parallel they must intersect each other in a point in ground line, since the coordinate planes and the given plane from a solid angle whose vertex is the point of intersection or piercing-point of ground line on that plane-points : gx, gy and gz."

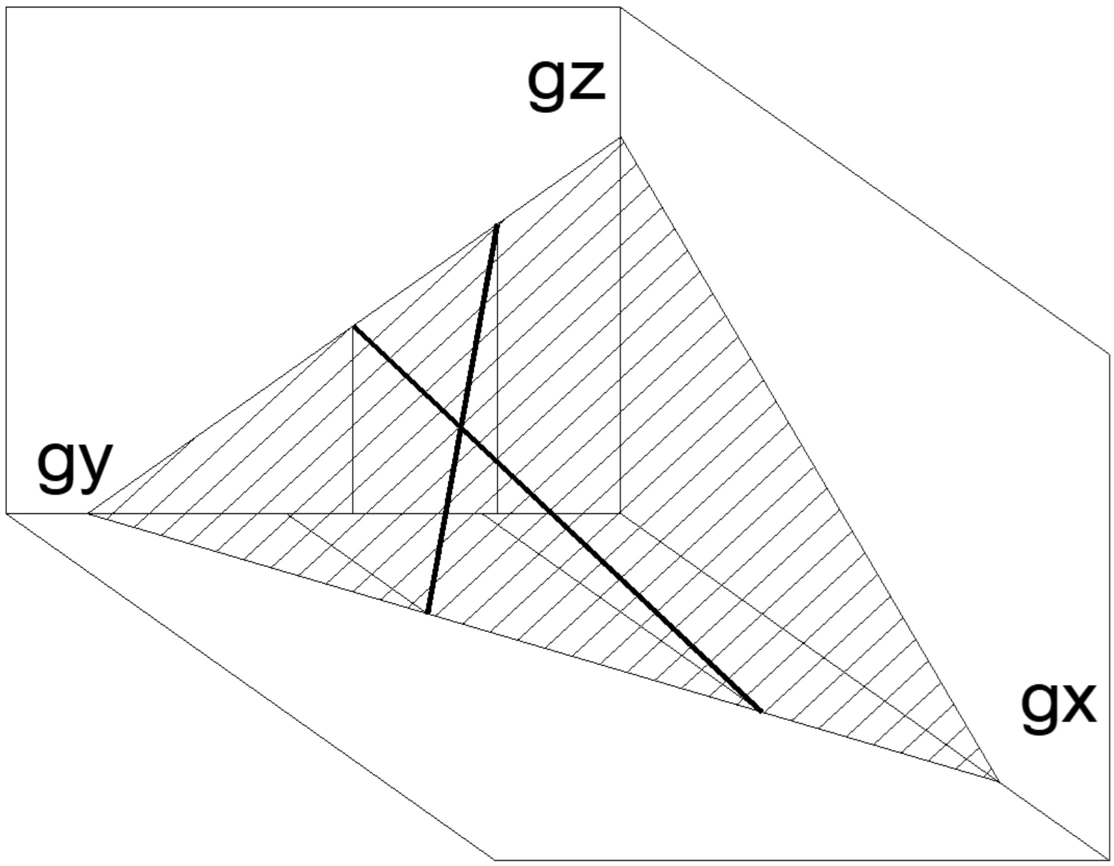


Figure 45: source author

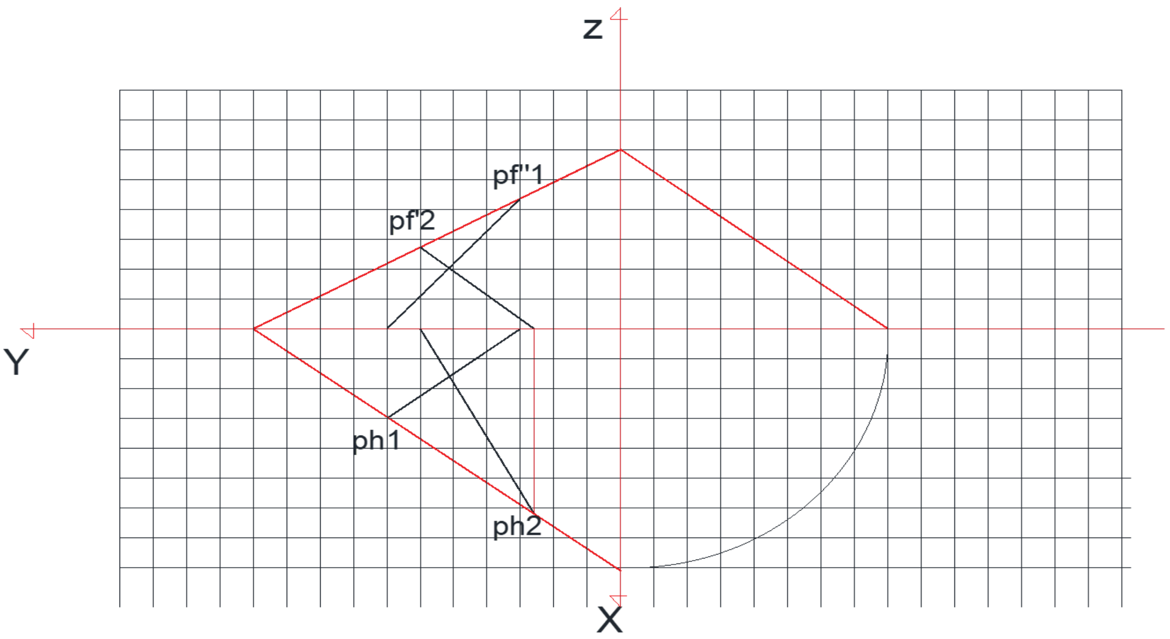


Figure 46: source author

.1.2 Remarkable planes

Some position of plane to the coordinate planes:

-a/ perpendicular to the horizontal projecting plane:

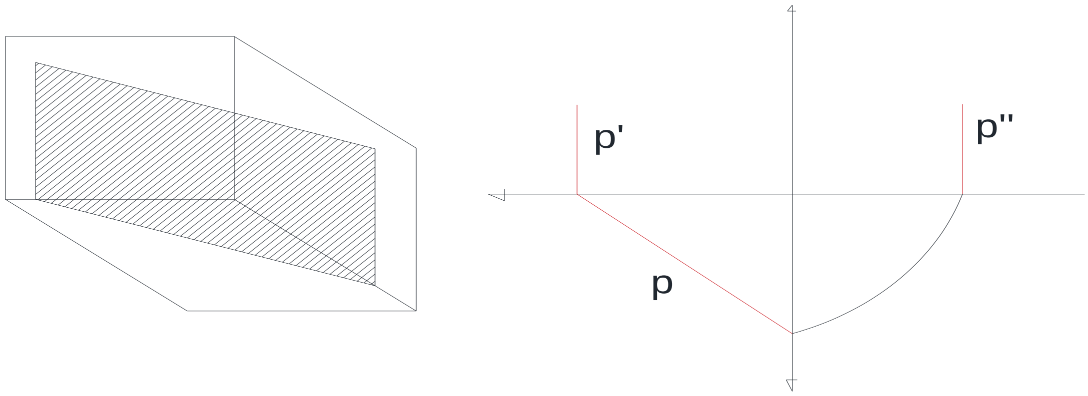


Figure 47: source author

b/ perpendicular to the frontal projecting plane:

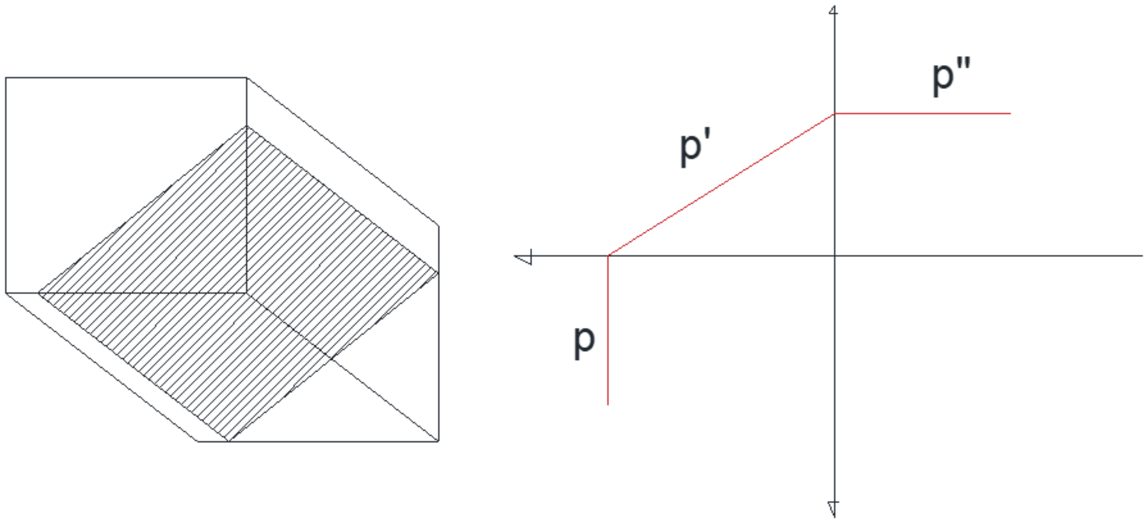


Figure 48: source author

c/ perpendicular to the profile projecting plane:

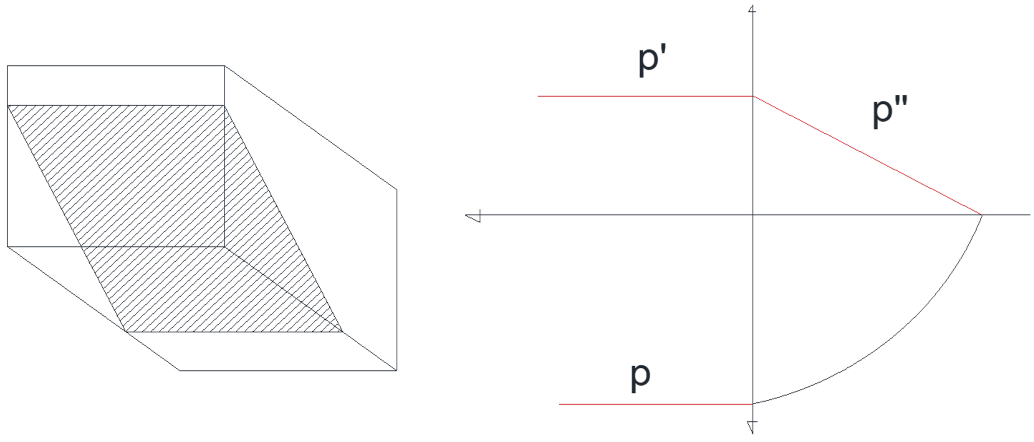


Figure 49: source author

d/ parallel to the horizontal projecting plane:

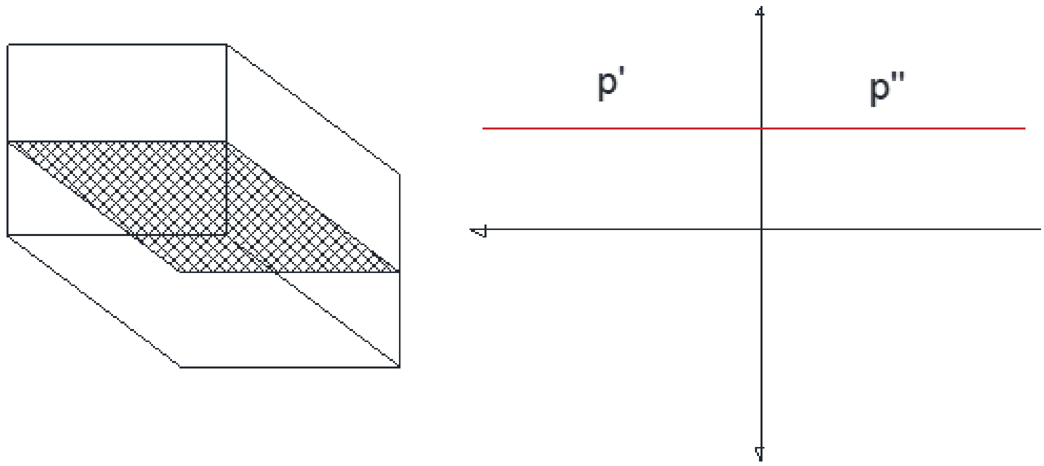


Figure 50: source author

e/ parallel to the frontal projecting plane:

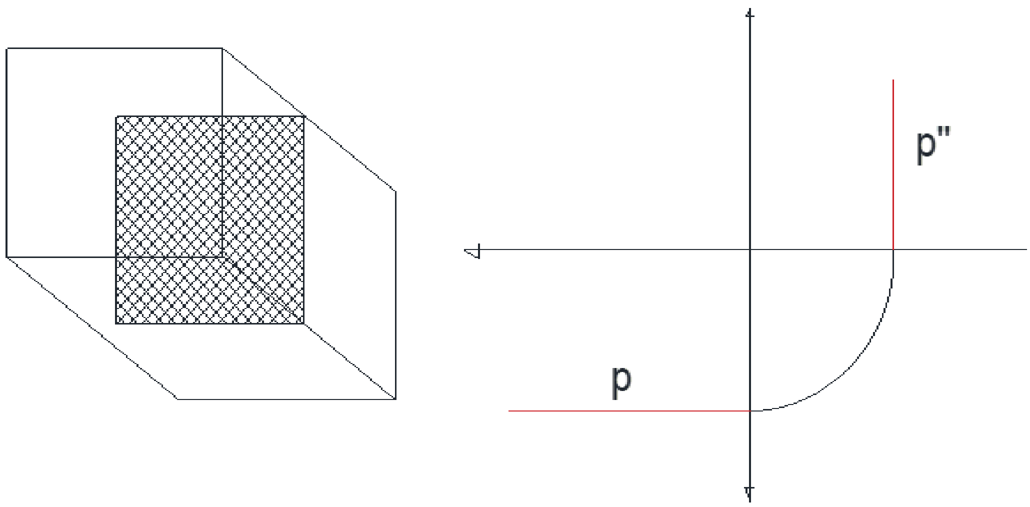


Figure 51: source author

f/ parallel to the profile projecting plane:

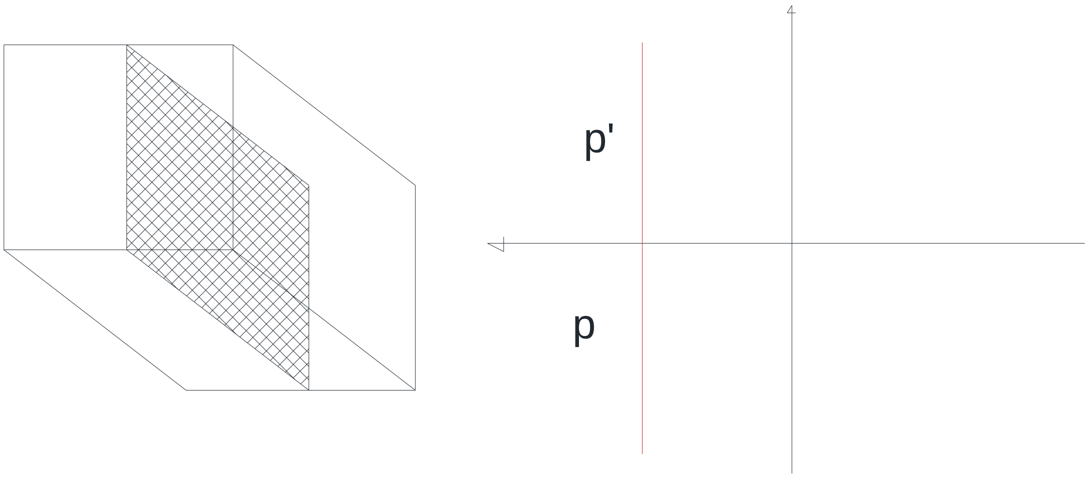


Figure 52: source author

j/ planes passes through the axis $\vec{y}$, $\vec{x}$ and $\vec{z}$

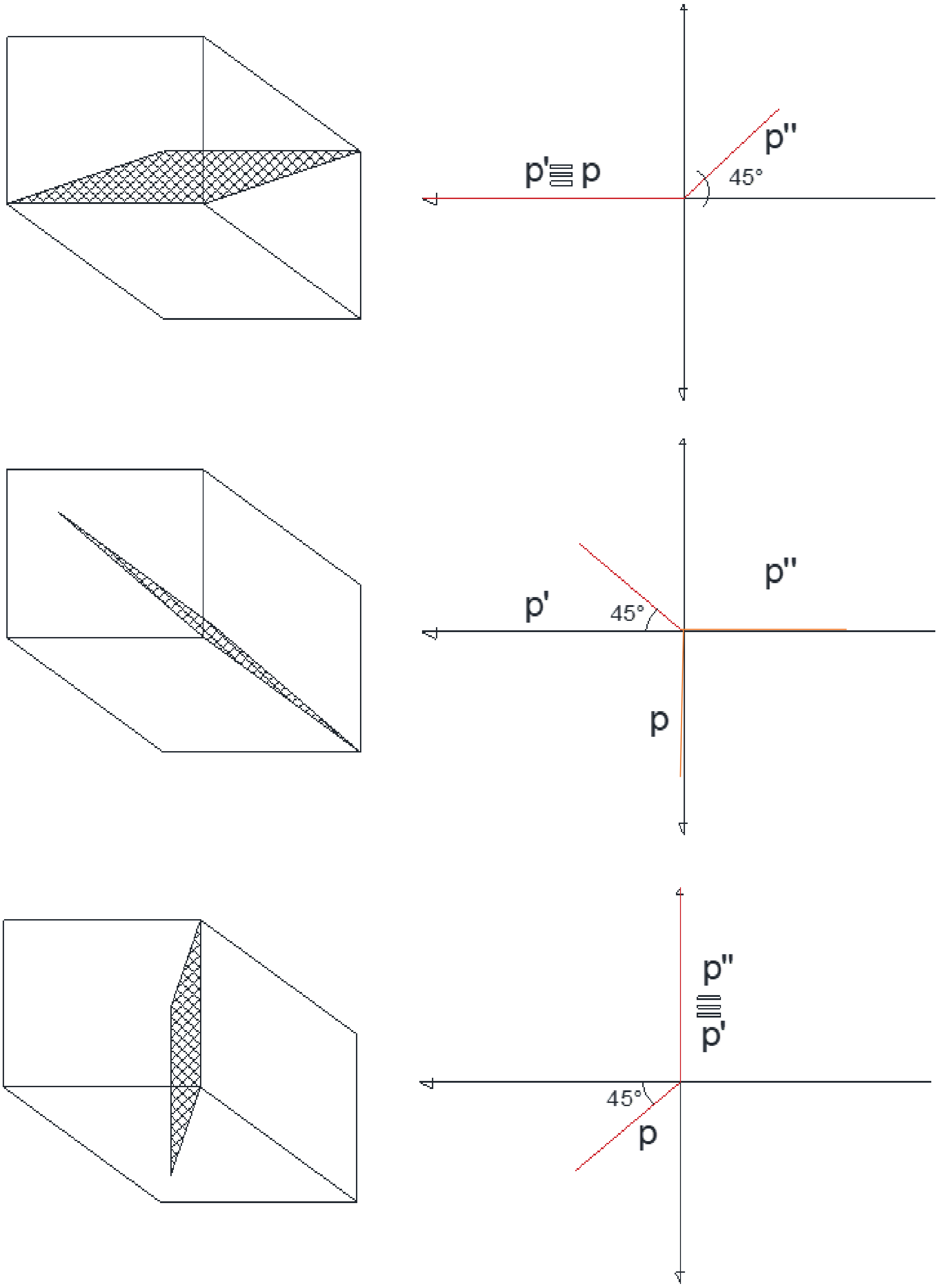


Figure 53: source author

.1.3 The intersecting planes :

The standard method for finding where two surfaces intersect involves using additional planes that cut through both surfaces, creating intersecting lines. These lines converge at a point that lies on both surfaces, determining their intersection line. When the surfaces are planes themselves, their intersection forms a straight line. Typically, only two of these auxiliary planes are needed to pinpoint this line accurately. This method capitalizes on the concept that when two common points are identified on intersecting planes, a line can be drawn between them. Employing this approach allows us to pinpoint the intersection line between planes P and Q.

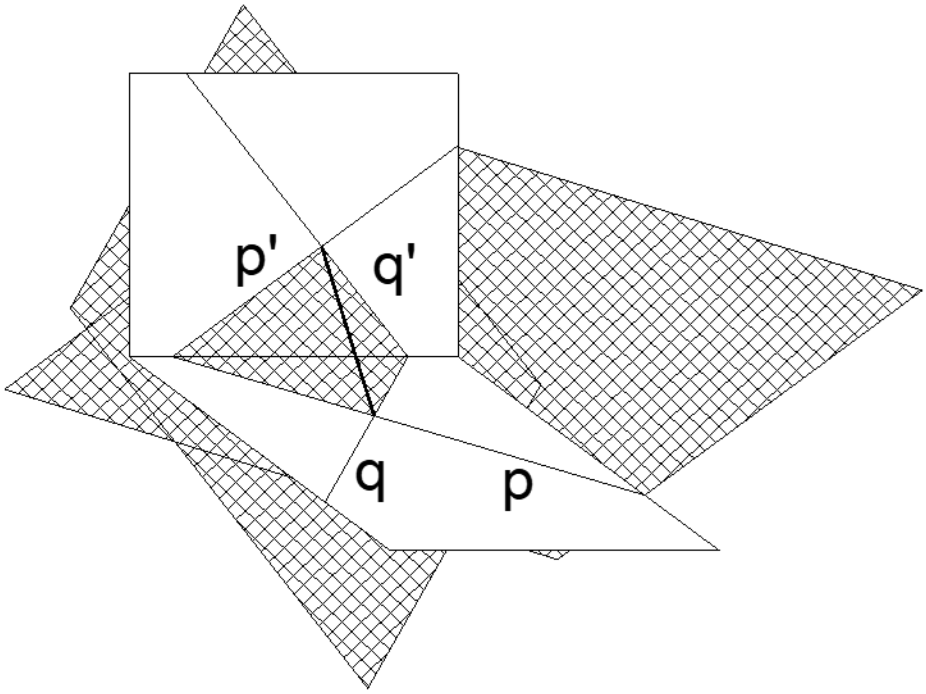


Figure 54: source author

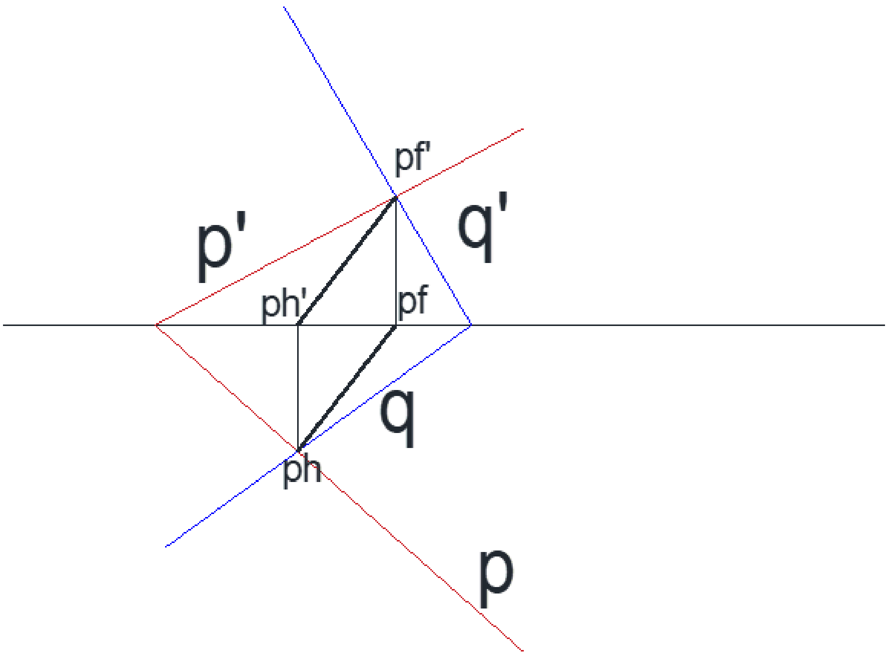


Figure 55: source author

5-The parallel and perpendicular planes :

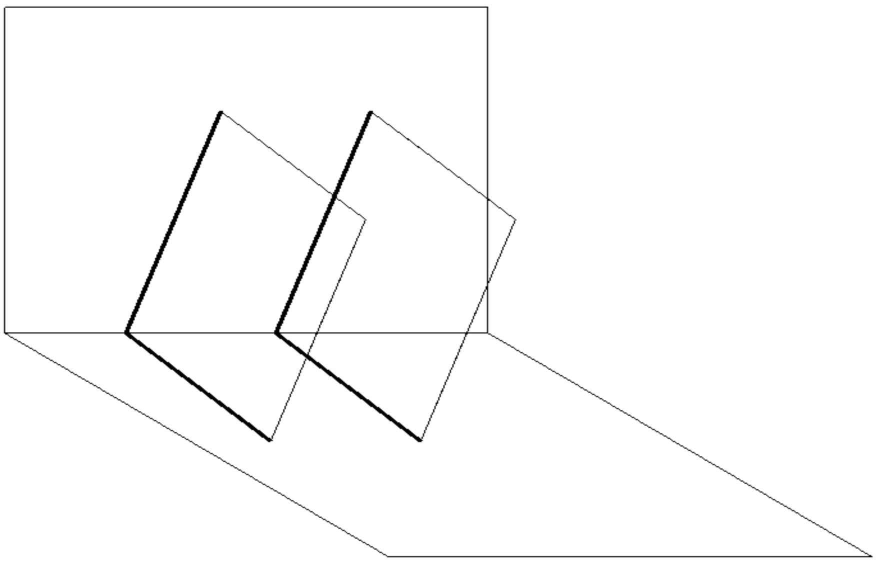


Figure 56: source author

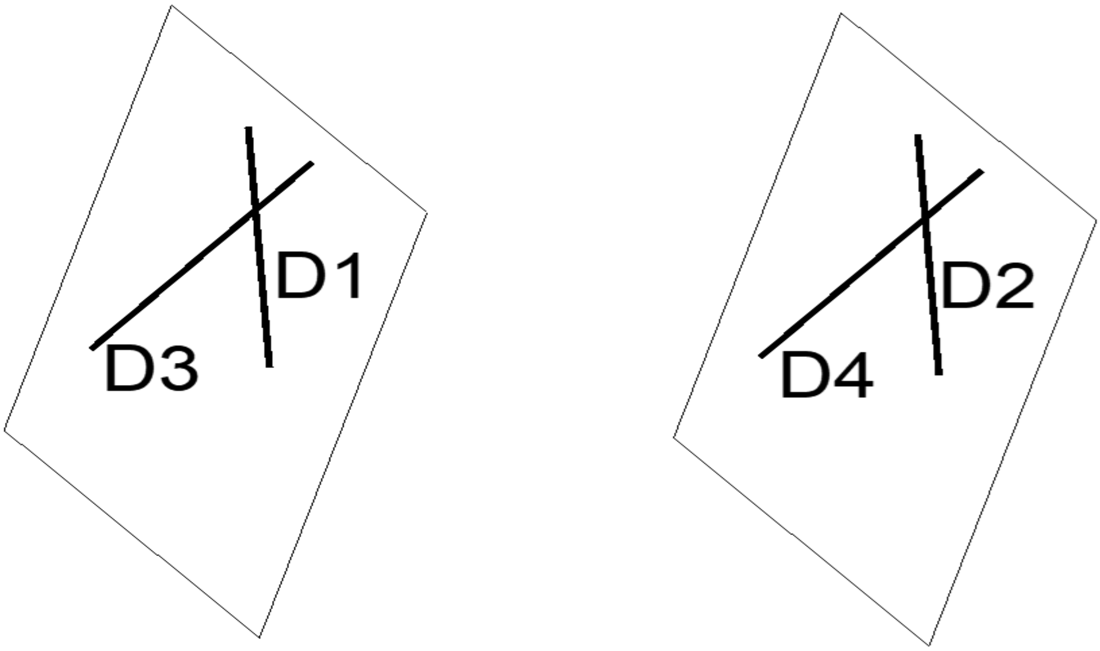


Figure 57: source author

The planes are parallel if they each contain two parallel lines. In simpler terms, if the projections of lines from one plane are parallel to those from another, the planes are parallel. This condition is confirmed by checking if their corresponding traces are also parallel

Exercice:

- Determine the traces of plane P, given two parallel lines D1 and D2 ? Determine the traces (b, b', b '') of the second upright plane (**perpendicular to the frontal projecting plane)** defined by points tf1 (+6. +6. 0) and tf2 (+3. +2.0) ? Find the intersection of the two planes, resulting in line L ?

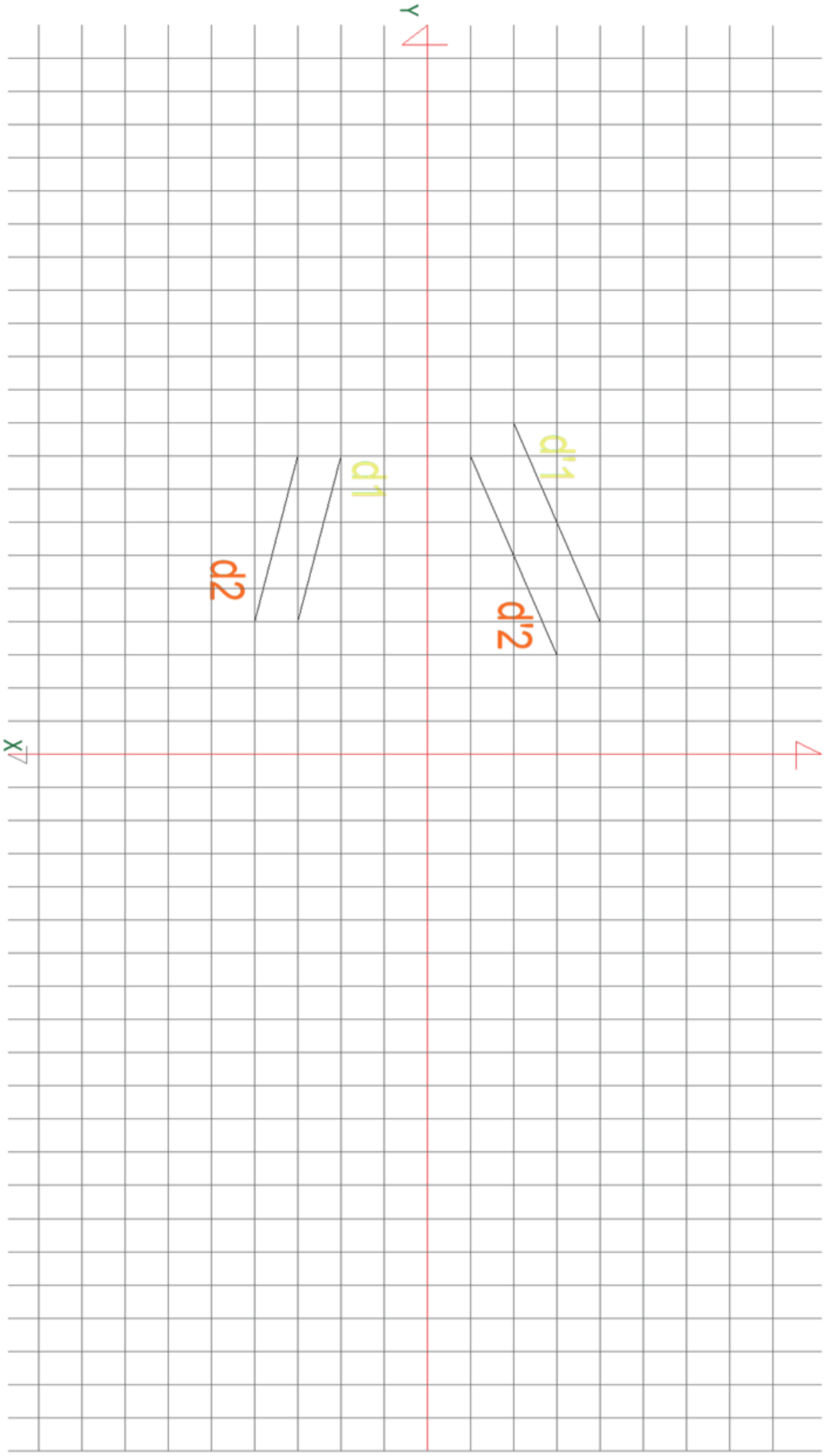


Figure 58: source author

-List the main methods for determining a plane.

-Illustrate three traces of a plane. How should they relate when projected?

-How are the traces of a plane related to a line lying in that plane? Validate with a representation.

-Define when a point lies in a plane. Provide a visual representation.

-Enumerate the primary positions of a plane relative to the coordinate planes.

-Explain the concept of "principal lines of a plane." Provide illustrations for any given plane.

-What is the outcome of the intersection of two planes? Show the resulting intersection assumed by traces.

-How is the point of intersection between a line and a plane determined? Outline the main stages of intersection determination.

-When is a line parallel to a plane?

-List scenarios when a line is perpendicular to a plane.

-Provide diagrams for two parallel planes.

-Enumerate the cases when two planes are perpendicular.

COURSE N-07: Drawing Transformation in geometry

1 Introduction

In geometry, the transformation of drawings plays a crucial role in accurately representing objects in space. This process enables us to determine their true sizes, shapes, and positions. Whether objects are parallel or perpendicular to coordinate or auxiliary planes significantly affects how we interpret their projections.

When an object aligns parallel to a plane, its projections on that plane accurately represent its true length. Conversely, when an object is perpendicular to a plane, its projections take on simpler geometric forms, such as points, lines, or planes.

There are five primary tasks involved in drawing transformation:

Defining the true length of a line involves determining its actual measurement without any distortion or approximation. This process is essential for accurate calculations and constructions. Transforming a line into its projected end view means representing it as it appears when viewed from one of its ends, providing a different perspective that may aid in visualization and analysis. Defining the true shape of a surface plane entails accurately describing its geometric form without distortion or simplification, crucial for various engineering and design applications. Transforming a surface plane into its projected edge view involves depicting it as seen from a specific viewpoint, highlighting its edges and boundaries for clarity and precision. Determining the distance between two skew lines requires calculating the shortest distance between them, considering their non-intersecting and non-parallel nature, which is essential in geometry and spatial analysis.

To address these tasks, various methods are employed:

1-Changing the projection plane (or ground-line).

2-Implementing parallel-plane transformation.

3-Utilizing rotation and rabattement techniques.

4-Employing rotation around plane traces, also known as the coinciding method.

Each method offers unique advantages and applications, which we will explore in more detail.

2 Changing the projection plane (or ground-line) / (Auxiliary View Method) :

The Auxiliary View Method is extensively utilized to determine the true size and shape of geometric objects and to project them accurately. In alternative resources, this approach is referred to as the Change of Position Method or the Auxiliary View Method. According to this technique, the object's points remain stationary, and their positions do not change. Instead, an auxiliary plane is introduced into the coordinate system. The placement of this auxiliary plane varies based on the specific task at hand. When defining the true length of objects, planes parallel to the objects and perpendicular to horizontal or vertical planes are employed (figure :60).

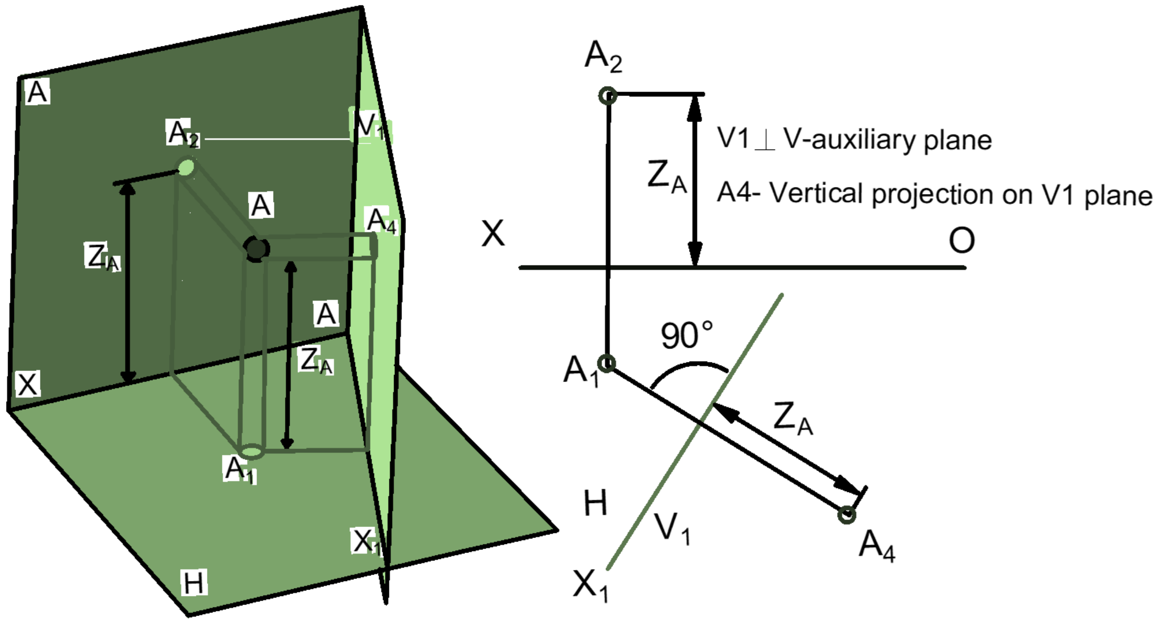


Figure 59: source author

In the new system H/V1, a modification is made by introducing the auxiliary plane V1 perpendicular to the H-plane, replacing the original vertical plane V. This alteration forms intersecting planes V1 and H, creating a new ground-line OX1. The projections of point A in this new system align along a corresponding line perpendicular to the OX1 projector of the planes H/V1. Notably, the distance of point A from the H-plane remains unchanged, ensuring consistency in the drawing process.

This condition holds significant importance in drawings. Introducing the new auxiliary plane perpendicular to the vertical plane ensures that the distance between the new plane and point B equals the distance between it and V. Consequently, if B1 and B2 represent the initial projections of point B and they intersect the new ground-line OX1, a perpendicular line B2 B4 is drawn, offsetting a distance equal to B0 B1 above OX1. Thus, point B4 represents the sought-after new horizontal projection.

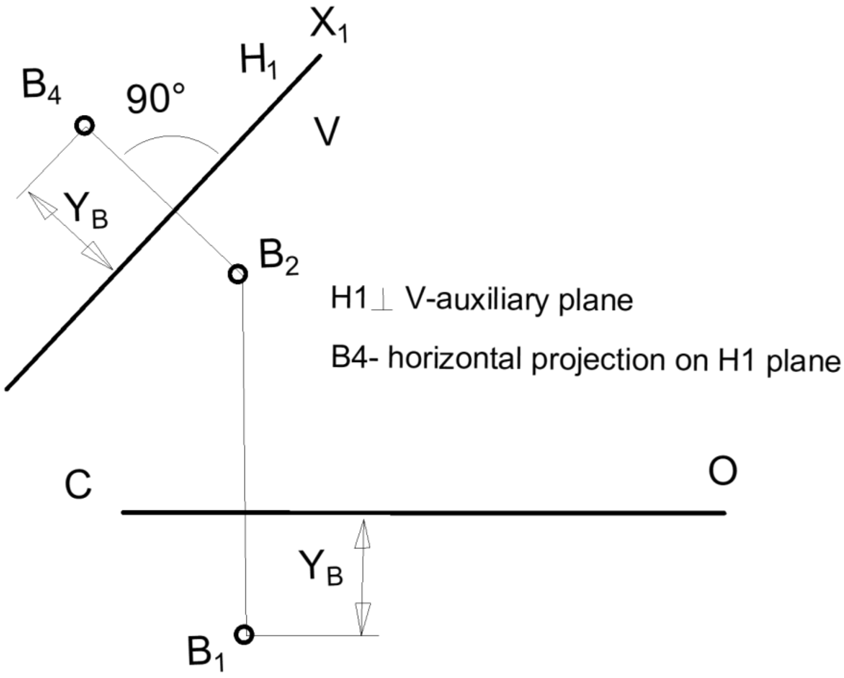


Figure 60: source author

<u>Defining the True Length of a Line</u>

The true length of a line is determined by projecting it onto a plane parallel to the line itself. The auxiliary plane, crucial for this projection, must run parallel to the line and be perpendicular to one of the coordinate planes. In this case, the new plane, P4, aligns parallel to line AB (its horizontal projection) and perpendicular to the H-plane.

Transforming the Line into its End View (figure62)

To achieve this transformation, perpendiculars are drawn from the projections of points A1 and B1 onto the new ground-line, OX1. The distance between these points and their corresponding vertical projection is marked off. The resulting line, A4 B4, represents the true length of AB, with the angle between this line and OX1 indicating its true angle relative to the H-plane.

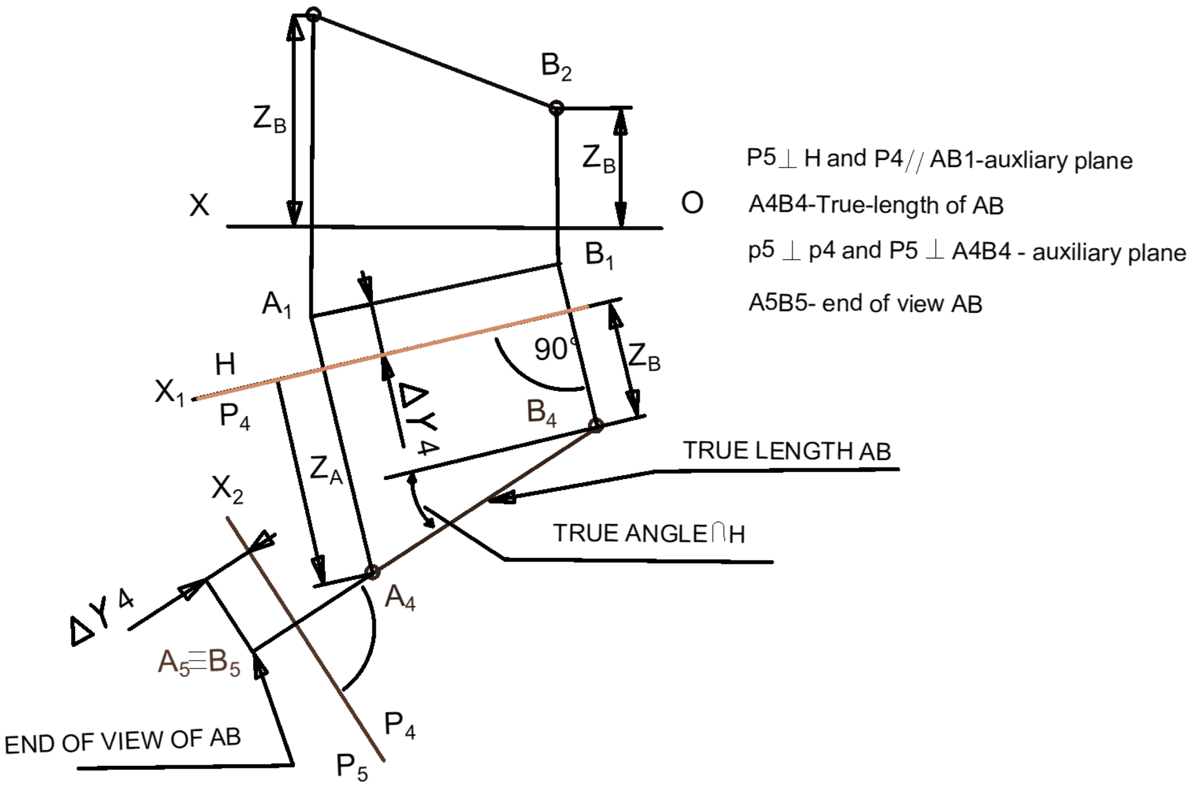


Figure 61: source author

Determination of the True-Shape and Edge View of a Plane

Simplified Position of the Plane A plane achieves its simplest orientation concerning the Horizontal (H) and Vertical (V) planes when it aligns parallel to one plane and perpendicular to the other. In this configuration, one projection serves as a normal view, while the other functions as an edge view.

Normal View and True Shape When the plane is depicted as a polygon, its normal view accurately portrays the true shape of the figure. This representation provides crucial insights into the geometric characteristics of the plane

Obtaining Edge and Normal Views When the plane deviates from the simple parallel-perpendicular orientation relative to the H and V planes, auxiliary planes come into play. By strategically selecting appropriate auxiliary planes, it's possible to obtain both the edge and normal views of the plane, even when it's inclined.

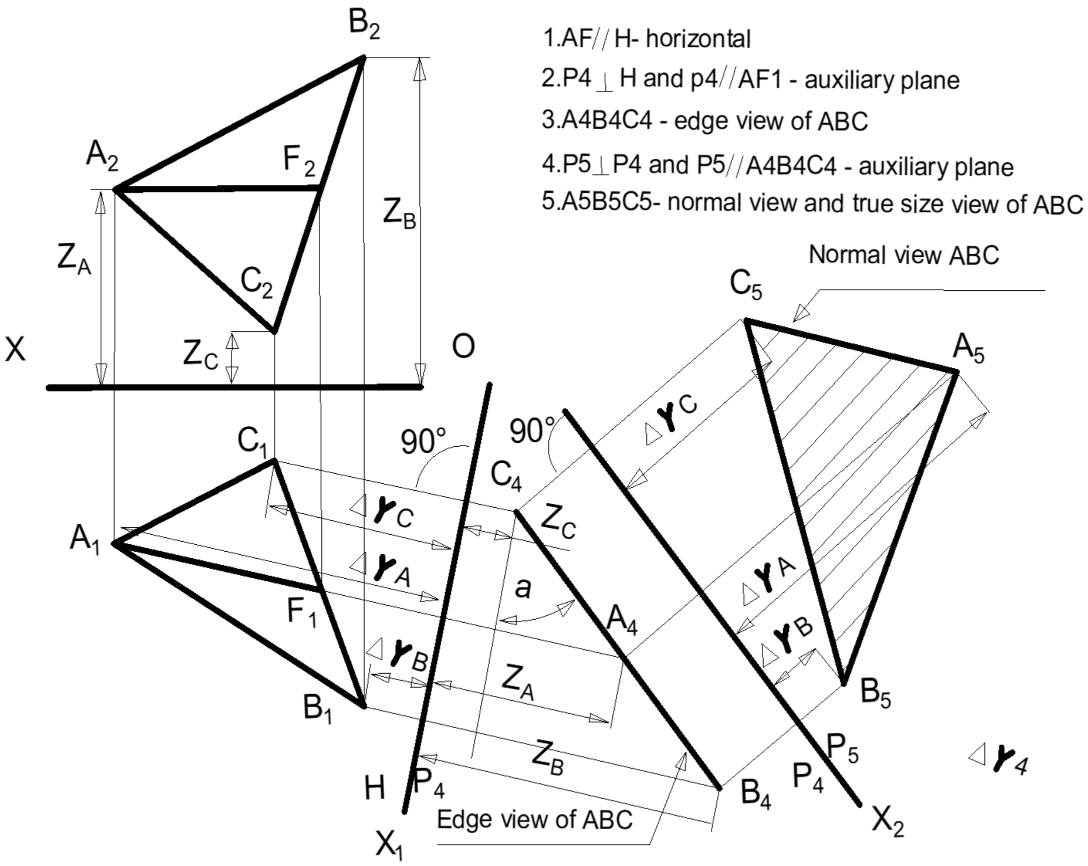


Figure 62: source author

The triangular representation of plane ABC is illustrated in Figure 63. Initially, the edge view of ABC is established.

-Plane Projection

It's essential to note that a plane projects as a straight line onto a projection plane perpendicular to any line within the plane. For convenience, the projection plane aligns perpendicular to a principal line of the plane.

-Auxiliary Line and True-Length View

An auxiliary line AF is drawn parallel to the horizontal plane-horizontal of ABC. Subsequently, A1F1 represents the true-length view of AF.

-Projection for Edge View

Plane P4, perpendicular to line AF, is established by positioning OX1 at a right angle to A1F1. Projecting ABC onto P4 yields the edge view ABC. The angle 'a' between A4B4C4 and OX1 corresponds to the angle between plane ABC and the horizontal plane H.

-Edge View on a Third Plane

Additionally, the edge view of ABC can be depicted on a third plane perpendicular to a frontal line of ABC.

-Normal View and True-Shape View

The normal view of a plane materializes on a projection plane parallel to the given plane. Plane P5, parallel to ABC, is created with OX2 drawn parallel to A4B4C4. Projecting ABC onto P5 (A5B5C5) presents the normal view of the plane and the true-shape view of triangular ABC.

Determination of the shortest distance between two skew lines.

Using the principle of the method, the common perpendicular can be drawn in the view in which one of the given lines projects as a point (end view of line). The distance between the lines is equal to the true-length of MN(Figure64).

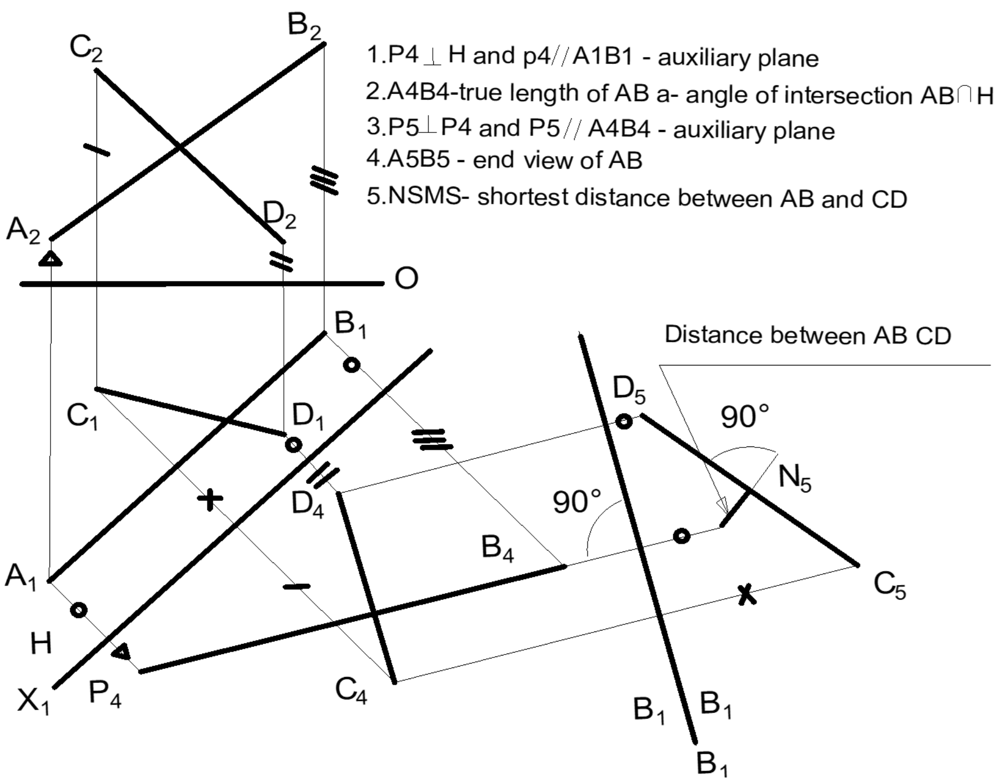


Figure 63: source author

3 Methods of rotation

Methods of rotation involve an alternative approach to altering the perspective of an object without changing its position within the projection plane. Instead of adjusting the plane of projection, new planes are introduced into the existing system to achieve the same outcome. This method enables the generation of different views and projections by simply repositioning the object itself. The operations involved in this process include movements either parallel to a rectilinear or plane direction, or rotations. It's essential to bear in mind that the primary goal is to simplify the construction process to facilitate problem-solving. Therefore, regardless of the alteration in position, the focus should always remain on streamlining the constructions for efficient solutions.

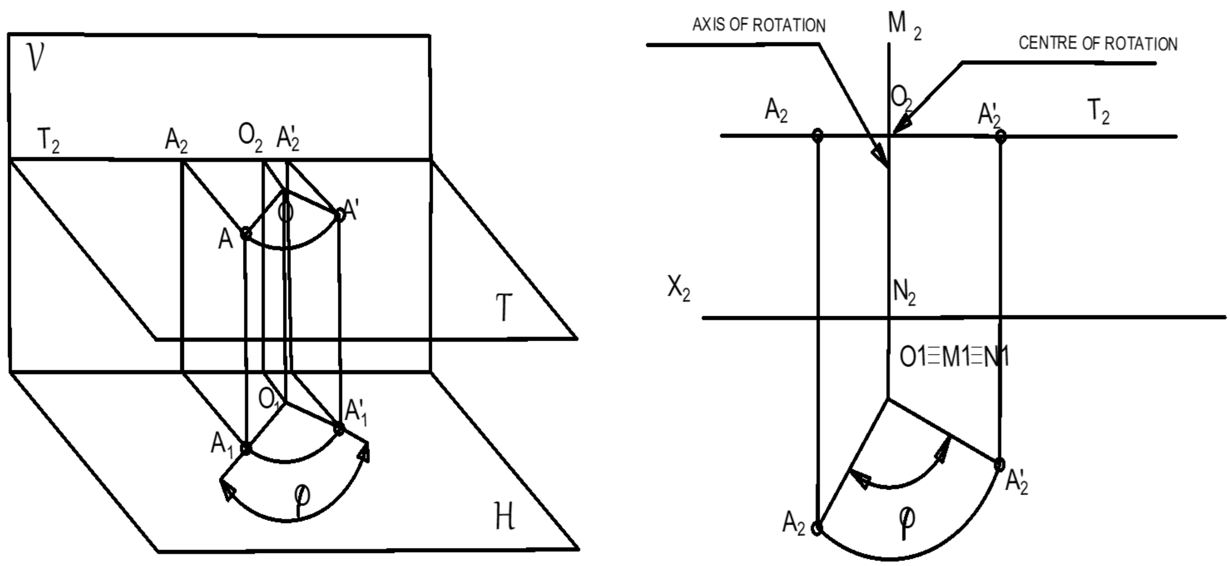


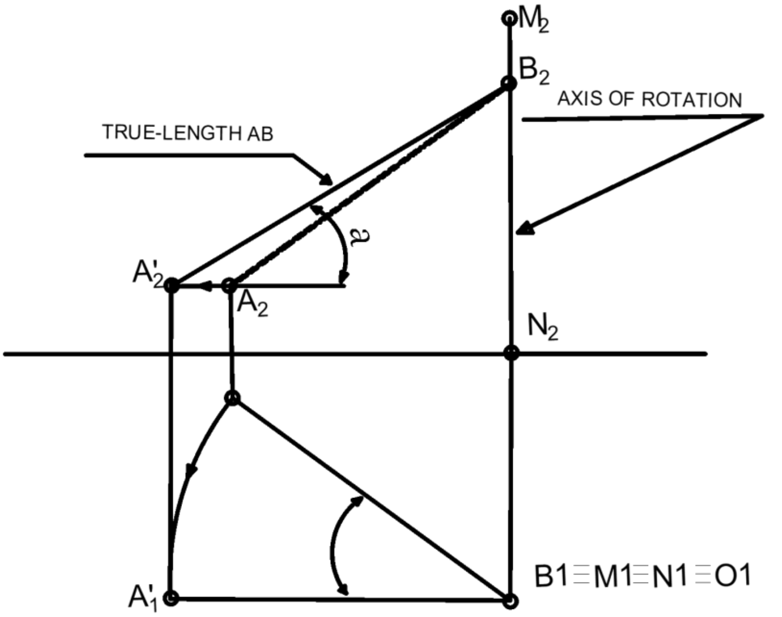
Figure 64: source author

The process of rotation inherently involves several key elements:

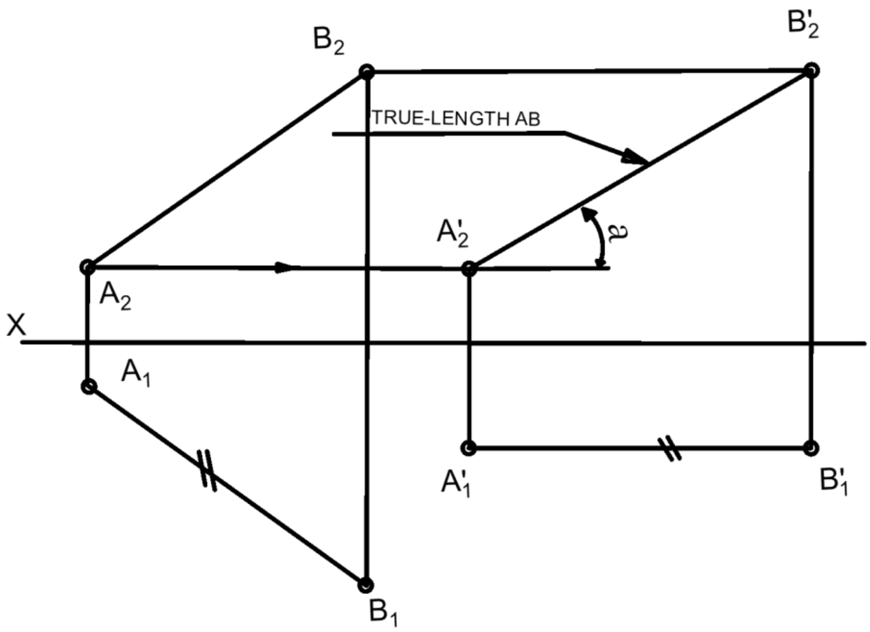
1. Axis of Rotation: Illustrated by the line MN, it serves as the pivotal point around which the object rotates. Understanding the orientation of this axis is crucial for accurate rotation.
2. Radius of Rotation: Denoted by AO, it represents the fixed distance of each object point from the axis throughout the rotation.
3. Center of Rotation: Marked by O, it signifies the point where the radius intersects the axis, serving as a stationary reference point.
4. Circle of Rotation: Represented by AA, it delineates the path traced by each point of the object during rotation.
5. Plane of Rotation: Depicted by T, it is always perpendicular to the axis, guiding the circular motion of points along the object.
6. Arc of Rotation: Defined by AA’, it describes the trajectory through which any point rotates, providing the angle of rotation for each object point

relative to the center of rotation A1O1A’2. These elements collectively define the rotational motion and orientation of the object

There are two general positions which such an axis may assume to the moving object:



Around a axis



Parallel plane method

Figure 65 : source author

The typical positions of an axis in relation to different geometric elements. For instance, with lines, the axis usually intersects at a point; with planes, it intersects at a point or line; with plane figures, it aligns with an axis, element, or tangent. In the second case, the axis tends to be parallel or in the same plane as the given element. It's essential for practical purposes to position the axis perpendicular to the projection planes to simplify constructions. To rotate a point around an axis, a perpendicular line (radius) is drawn through the point to the axis, with the original distance marked off. Rotation around a line perpendicular to the projection plane is common for defining true line lengths. Projections on a plane perpendicular to the axis remain unchanged in size and shape, while others alter along parallel axes of coordinate planes. These principles apply in parallel-plane transformation, where one projection is moved without size changes to align with a chosen plane projection, and subsequent projections are drawn parallel to the X-axis.

4 Rabbatement:

The method of rotation, known as rabbatement, is employed to avoid constructing objects in ways that may cause confusion due to their position in space. Additionally, it's often necessary to precisely determine the size, shape, and position of objects or their components in practical scenarios

1. Application to Plane Objects: Consider a scenario where the rotating object is a plane. If this plane rotates around a line parallel to either of the coordinate planes until it aligns with that plane, or if it rotates around a line within a coordinate plane until it coincides with that plane, the rotation is achieved through rabbatement. During this process, rotation occurs around the horizontals or verticals of the plane until it becomes parallel to the corresponding horizontal or vertical plane
2. Determining the Radius Length: It's crucial to ascertain the true length of the

radius of rotation using various established methods, such as right-angle triangular calculations, rotational techniques, or parallel-plane transformations

This approach ensures accurate representation and positioning of objects or their parts in space, enhancing clarity and precision in construction and analysis.

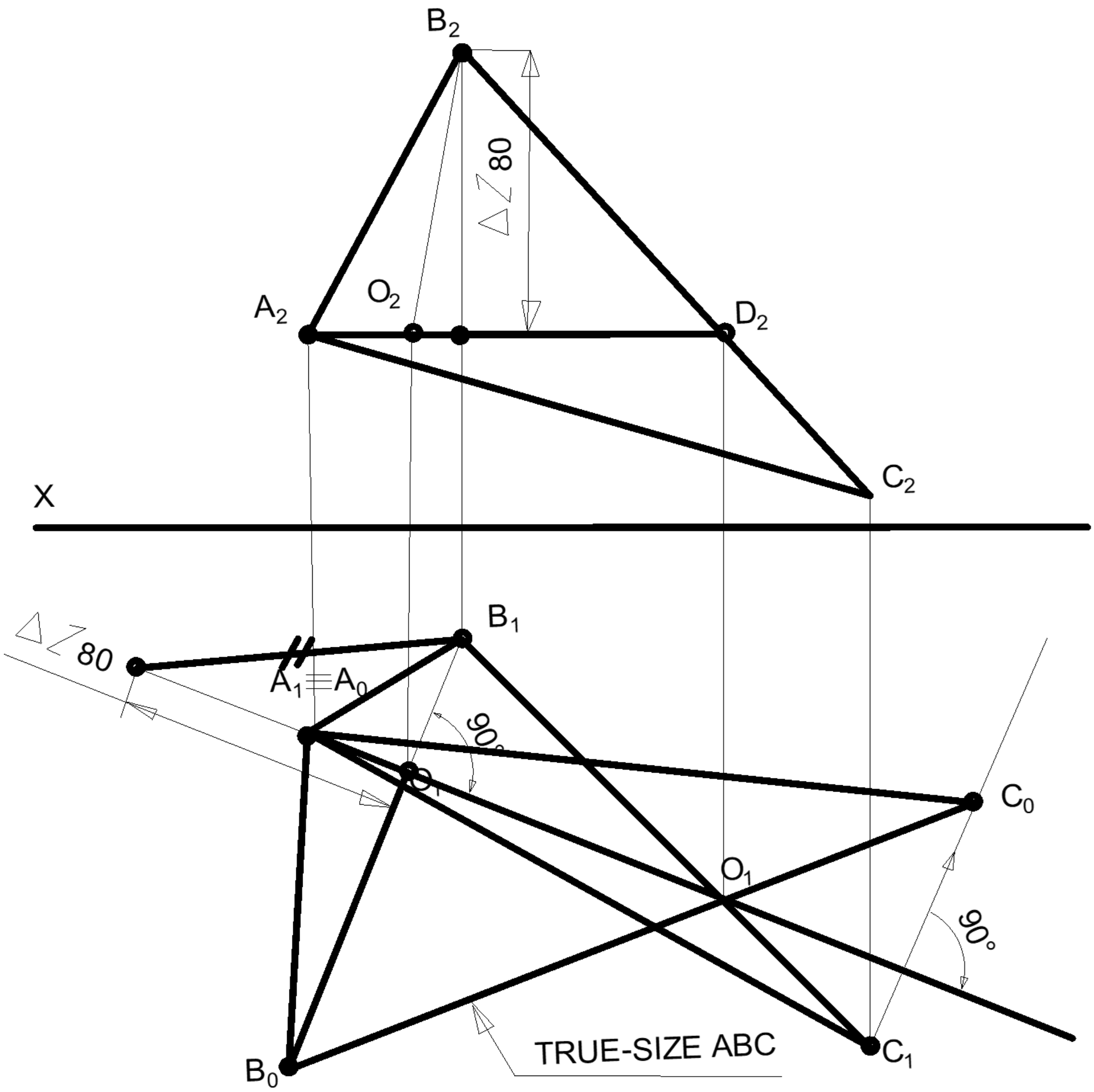


Figure 66: source author

3. The true size of a triangle is determined using the method of rabbatement, which involves revolving it around the horizontal line AD. As the triangle

rotates around this horizontal axis, perpendicular lines are drawn through points B1 and C1 to the horizontal line A1D1, depicting their rotational path around the horizontal axis.

4. Next, the true length of radius B1O0 is calculated using the right-angle triangular method. By drawing the true length of OB through the center of rotation of point B, denoted as point O1, and establishing the new projection of point B, labeled as B0, the accurate radius length is determined.
5. To find the horizontal projection of point C0, the intersection point of line B0D1 and a perpendicular line passing through C1 is identified. Finally, the triangular shape A0B0C0 represents the true shape of the plane that parallels the horizontal coordinate plane.
6. This method ensures precise depiction and measurement of the triangle's dimensions and orientation in space

5 Rotation around plane traces (or coinciding method)

In the study of geometric transformations, particularly in cases where a plane is defined by its traces, a common method employed is known as the coinciding method. This method proves invaluable when determining the true length of a line, such as AB, situated within the plane T.

Firstly, the given plane undergoes rotation around one of its traces until it aligns with a coordinate plane. This rotational axis, denoted as T1, dictates the transformation process. As the plane rotates, each point within it, along with line AB, moves within planes perpendicular to the horizontal trace T1.

The rotation of a specific point, like point C, guides the establishment of a new vertical trace, T2. This rotation defines the radius of rotation, TxC2, which shares the true length of a line lying in the frontal plane. The center of rotation, TX, is pivotal in this process.

To determine the true length of AB, an arc with radius TxC2 is drawn through TX until it intersects the perpendicular line passing through point C1, marking C's path during rotation. The resulting new trace, T'2, facilitates the determination of A'2B'2, the true length of AB, by aligning horizontals parallel to trace T1.

Haut du formulaire

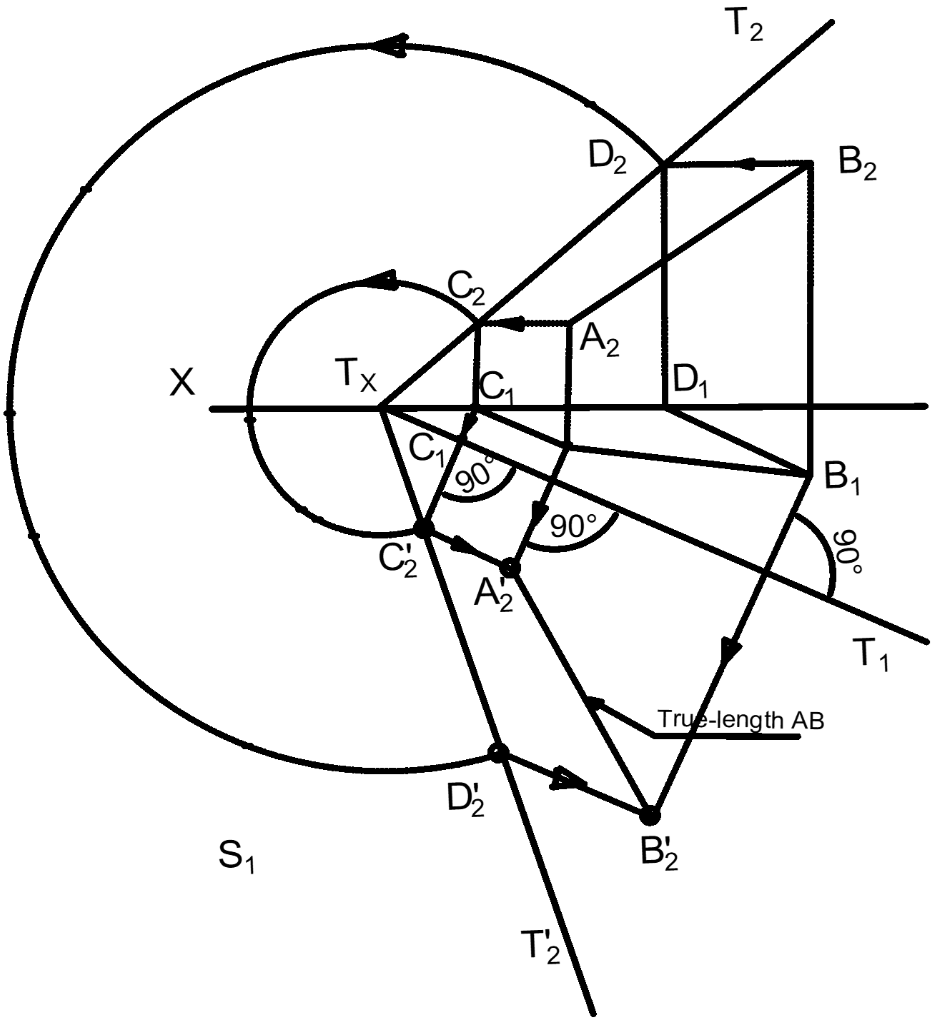


Figure 67: source author

Exercice:

1. Represent the rotation of a line and give the principles of parallel-plane method.
2. What does the term "rabbatement" mean? Give the principles of rotation by rabbatement for any line.
3. Describe the method of rotation around plane traces (or coinciding method).

COURSE N°08: plane surfaces (polyhedrons) and Representation of circle / cone / cylinder / sphere

1 Defenition polyhedrons:

Polyhedrons are solid objects bounded by planes. More precisely, it's the surface of the solid that constitutes the polyhedron. However, the term is commonly used to refer to either the surface or the solid itself.

Edges and Faces: The boundary planes of a polyhedron intersect to form its edges. A plane polygon formed by a set of edges constitutes a face of the polyhedron.

Vertices: The points where a set of faces intersect are called vertices. These vertices play a crucial role in defining the shape and structure of the polyhedron.

Regular Polyhedrons: A polyhedron is considered regular when all its faces are congruent. There are five regular polyhedrons: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. These solids possess symmetrical and uniform faces.

Irregular Polyhedrons: When the faces of a polyhedron are not congruent, it is classified as irregular. Examples of irregular polyhedrons include prisms, pyramids, and prismoids.

By understanding the properties and classifications of polyhedrons, we can effectively analyze and visualize various geometric structures in Descriptive Geometry.

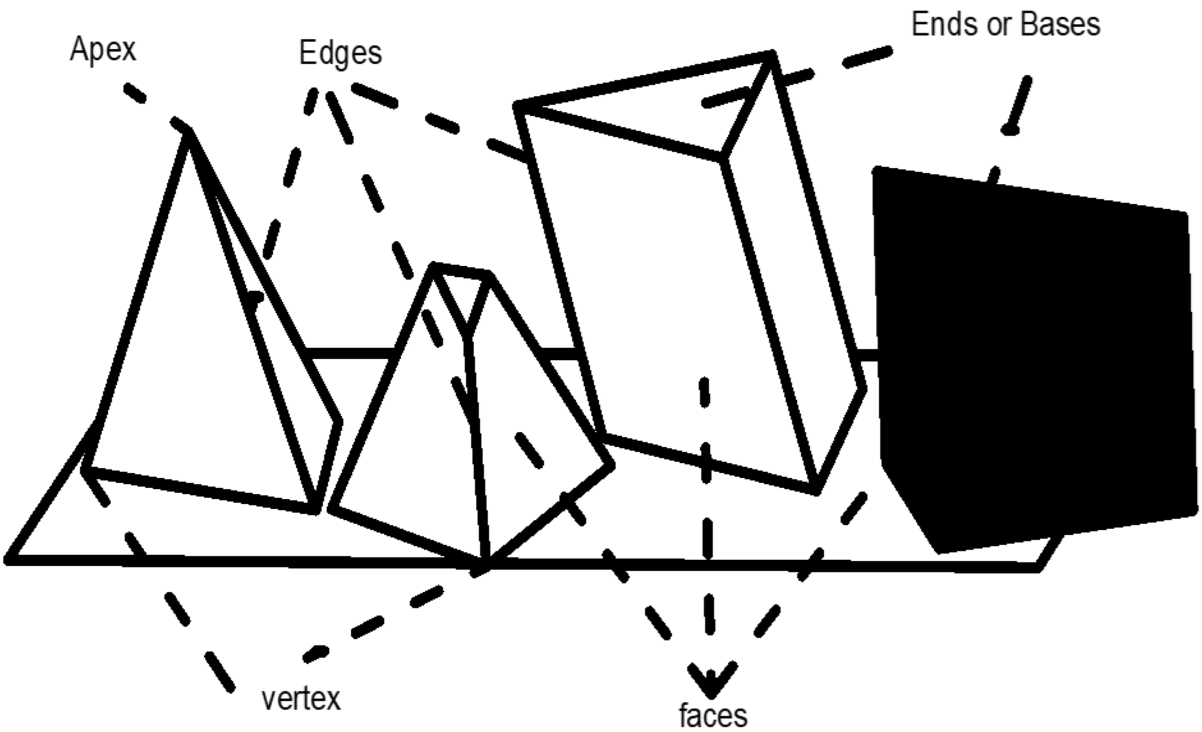


Figure 68: source author

2 Prisms, Prismoids, and Pyramids :

Let's explore the fascinating world of prisms, prismoids, and pyramids, fundamental geometric solids with distinct properties and characteristics.

2.1 Prism:

- A prism consists of two congruent and parallel polygons known as ends or bases.
- All other surfaces of the prism are parallelograms, termed as faces.
- When each face is a rectangle, the prism is a right prism; otherwise, it's oblique.

- An axis, parallel to one set of edges, connects the centers of the bases.
- The shape of the base defines the prism; rectangular bases result in a rectangular right prism, while parallelogram bases form a parallelepiped.

2.2 Prismoid:

- The prismoid features two parallel ends or bases with dissimilar polygons but having the same number of sides.
- Its faces are plane quadrilaterals.
- A prismoid can be visualized as the shape formed by connecting the corresponding vertices of the bases.

2.3 Pyramid:

A pyramid has a polygon for one face called the base, with all other faces being triangles.

The triangular faces converge at a common point known as the apex.

An axis connecting the apex and the center of the base defines the pyramid's orientation.

It's a right pyramid if the axis is perpendicular to the base; otherwise, it's oblique.

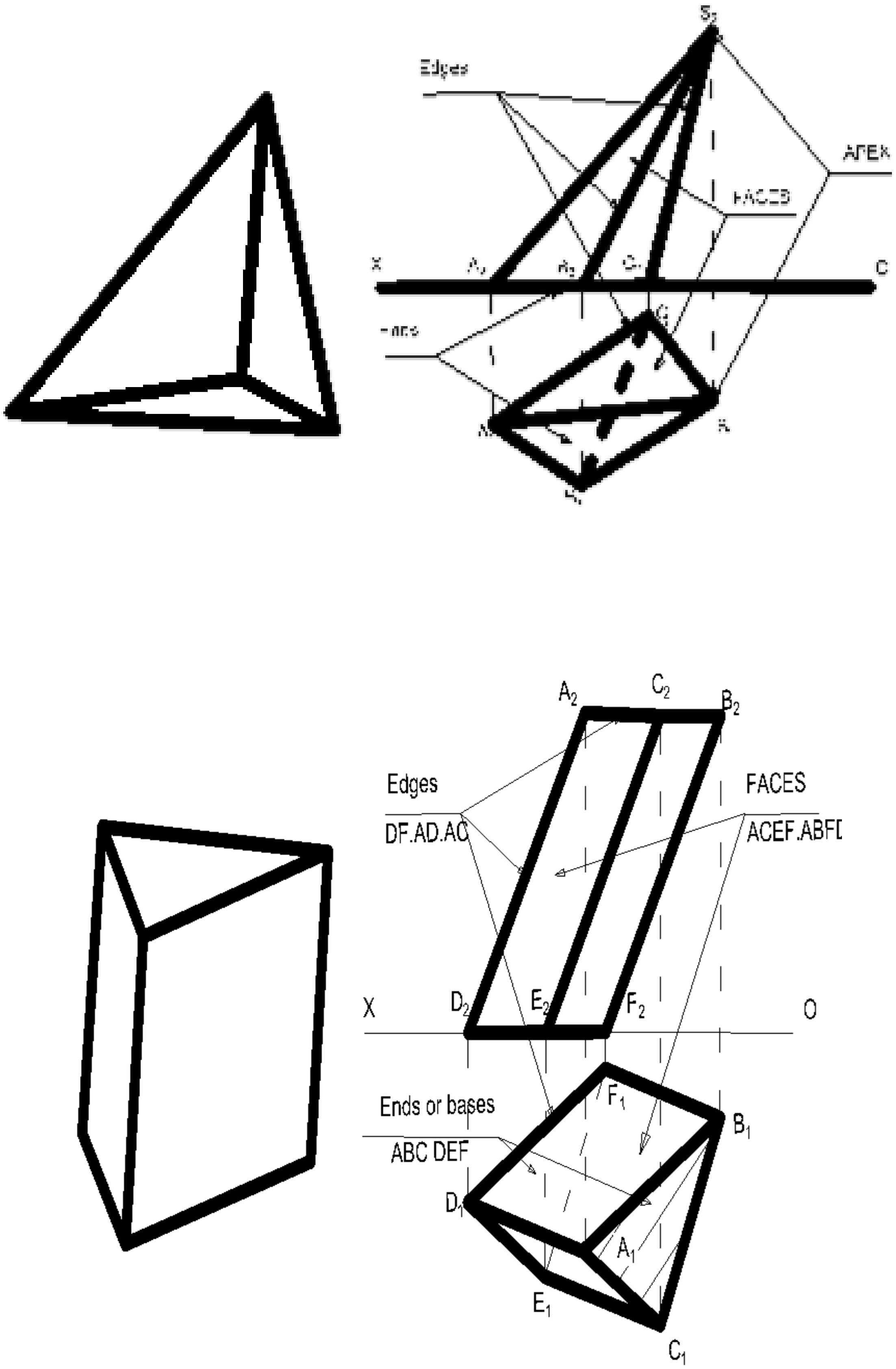


Figure 69: source author

3 Representation of cone / cylinder / sphere

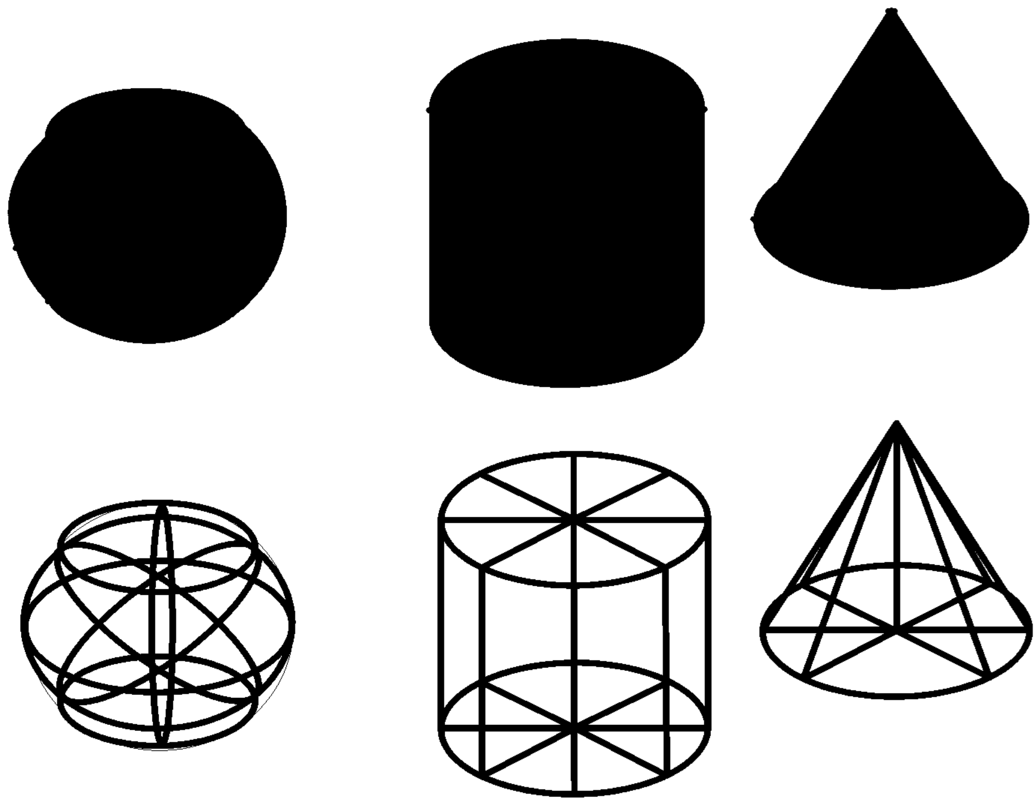


Figure 70: source author

Representation of Circle

A circle is a perfectly round shape with all points equidistant from the center. To represent a circle in Descriptive Geometry, we use its orthographic projections on different planes. The circle appears as an ellipse in both the front and top views, with its center projected onto each view.

Representation of Cone

A cone is a three-dimensional geometric shape with a circular base that tapers smoothly to a point called the apex. In Descriptive Geometry, we represent a cone by projecting its base and apex onto different planes. The base appears as a circle in both views, while the apex is projected as a point.

Representation of Cylinder

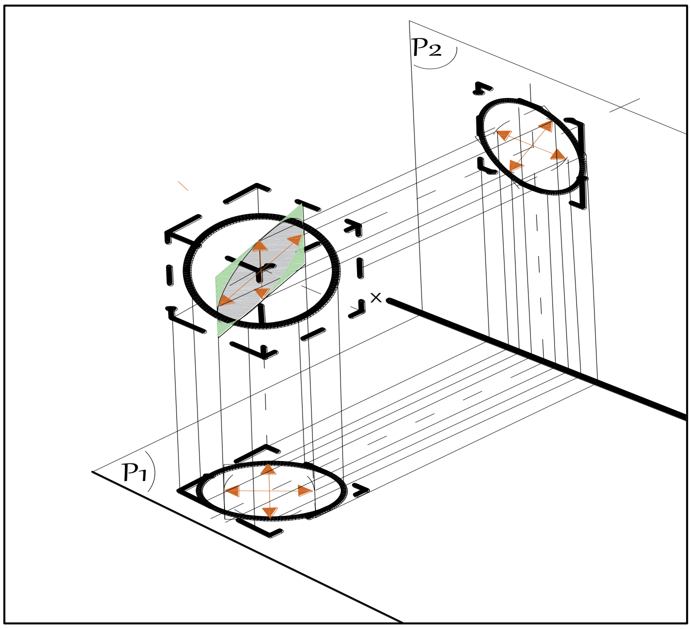
A cylinder is a three-dimensional shape with two parallel circular bases connected by a curved surface. When representing a cylinder in Descriptive Geometry, we project its bases onto different planes. Each base appears as a circle in both views, and the curved surface is represented as parallel lines connecting corresponding points on the circles.

Representation of Sphere

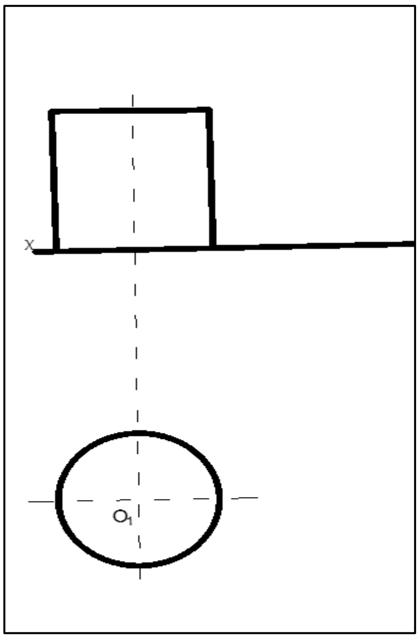
A sphere is a perfectly round three-dimensional object in which every point on its surface is equidistant from its center. To represent a sphere in Descriptive Geometry, we project its surface onto different planes. The sphere appears as a circle in both views, with its center projected onto each view.

In conclusion, Descriptive Geometry provides us with the tools to accurately represent complex three-dimensional objects on a two-dimensional plane. By understanding the principles behind the representation of circles, cones, cylinders, and spheres, we can effectively communicate and visualize these shapes in various engineering and architectural applications.

Now, let's explore some practical examples to solidify our understanding of these concepts.

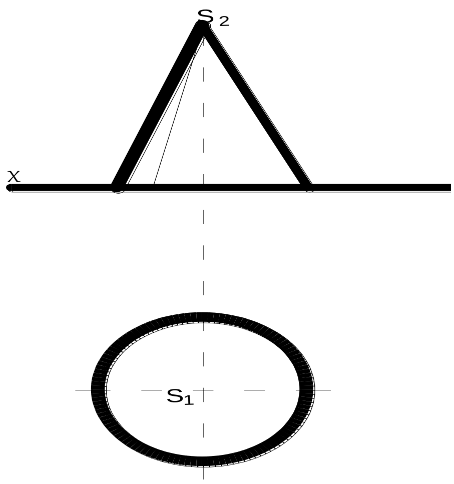


Representation of Sphere on a tree-dimensional



Representation of Cylinder on a two-dimensional plane.

Figure 71: source author



Representation of Cone on a two-dimensional plane.

Figure 72: source author

4 Intersecting polyhedron and plane:

In the study of plane-surface interactions, planes can assume two fundamental positions relative to a surface:

- **Tangent Plane:**
  - A plane can touch a surface, acting as a tangent. In this scenario, the entire surface lies on one side of the plane.
- **Secant Plane:**
  - Alternatively, a plane can cut through a surface, acting as a secant. This action divides the surface into distinct sections.

The intersection between the cutting plane and the surface is termed the Line of Intersection. This line is common to both the surface being cut and the cutting plane. Thus, irrespective of the surface's characteristics, the line of intersection remains a planar entity.

When examining the section of a polyhedron by a plane, it becomes apparent that the resulting section is entirely rectilinear and decreases in size as the plane approaches the vertex.

To determine the line of intersection, there are two primary methods:

a- Method of Edges:

- This method involves determining the intersection points of the secant plane with the edges of the polyhedron. It employs the fundamental rules of line and plane intersection.

b- Method of Faces:

- Here, the intersecting lines of the secant plane with the faces of the polyhedron are determined. The main principles of intersection between two planes guide this approach.

Understanding these concepts is crucial for analyzing geometric relationships and solving problems involving plane-surface interactions.

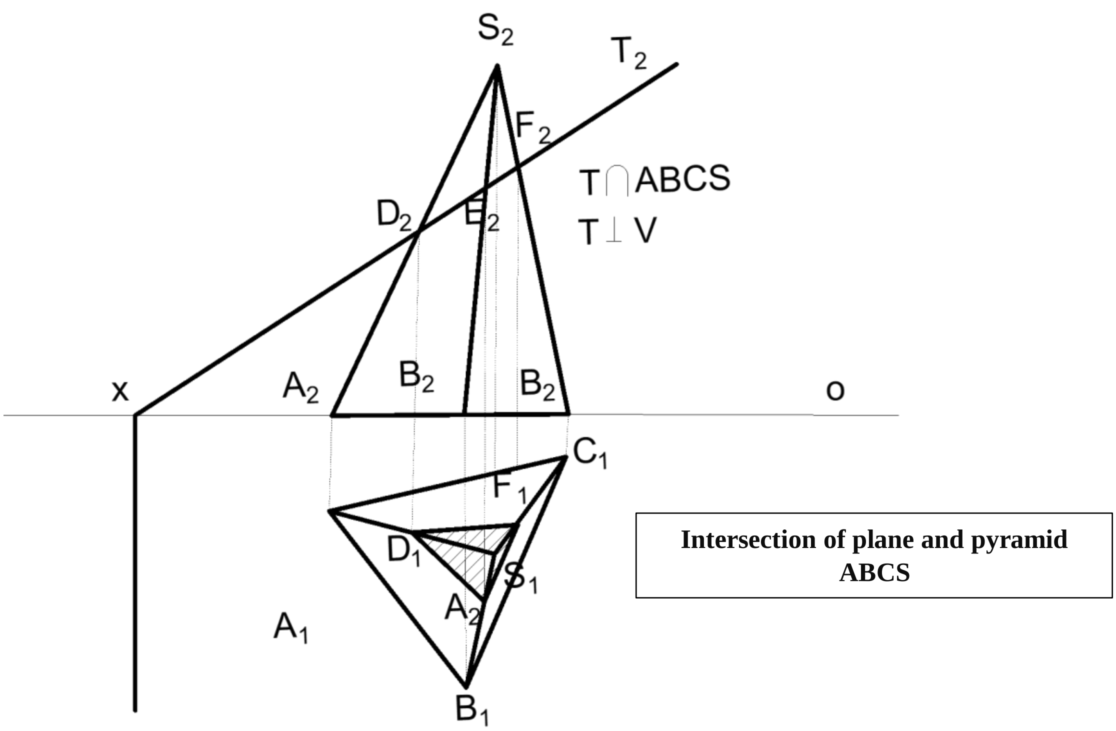


Figure 73: source author

5 Intersecting polyhedron and line:

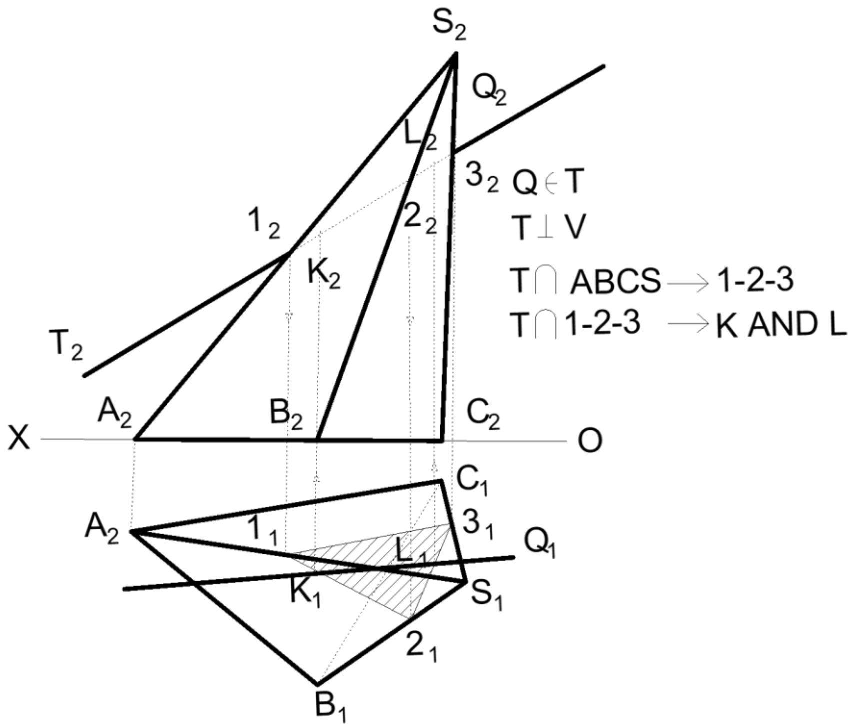


Figure 74: source author

We determine the points where the polyhedron (ABCS) intersects with the line (a) through the intersection of the line and plane (as illustrated in Figures 74, 75).

The line lies within the plane (T), which is perpendicular to one of the coordinate planes (a ЄT and T is perpendicular to V).

We establish the intersection between the plane and the polyhedron, specifically the section (1-2-3).

The points of intersection between the given line and plane are identified as the points where the line intersects the edges of the depicted section and the line itself (points K and L).

6 Intersecting of two polyhedrons:

When two polyhedra intersect, the line of intersection is determined by the points that result from the intersection of edges of one polyhedron with faces of the other (forming an intersecting line and plane). Another method involves defining the lines of correspondence between faces of the polyhedra (intersecting two planes).

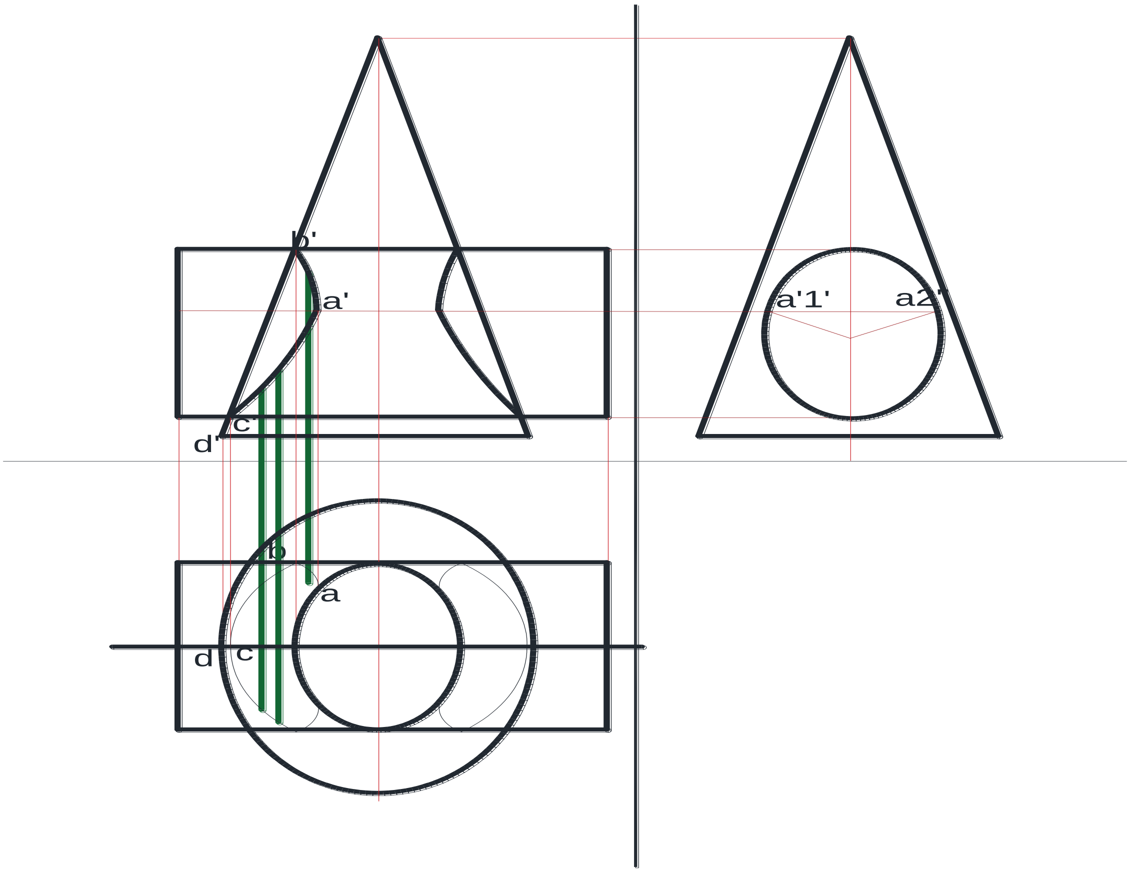
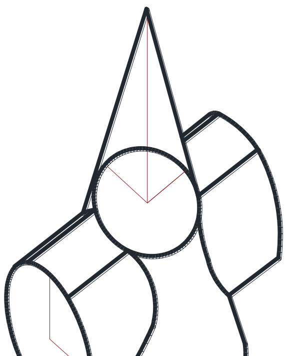


Figure 75: source: source author



- **What is a system of intersecting of two polyhedrons** ?

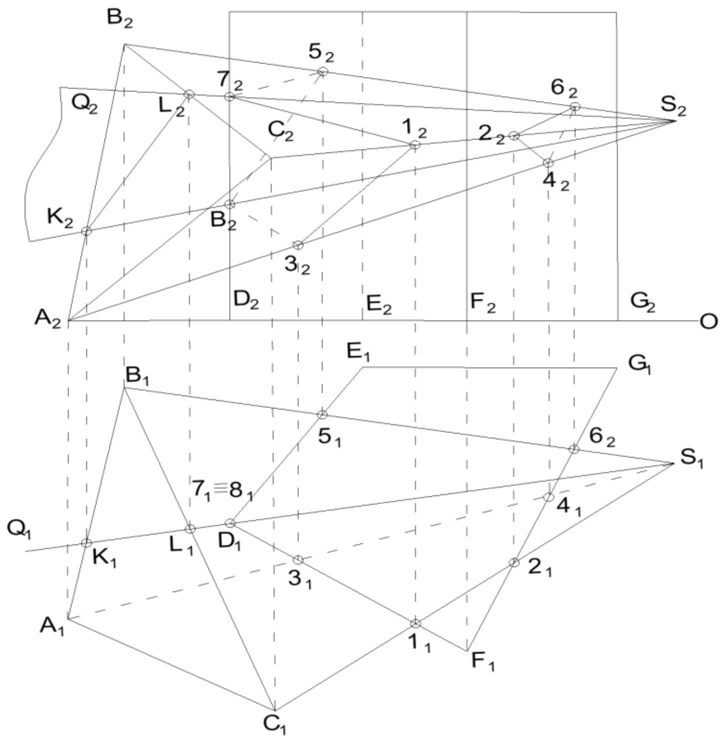


Figure 75: https://drawing-portal.com/en/zadachi-nachertatelnoj-geometrii.html

In Figure 76, we illustrate the result of intersecting a prism and a pyramid. Points 1, 2, 3, 4, 5, and 6 are determined by the intersection of the edges of the pyramid (AS, BS, and CS) with the faces of the prism. Points 7 and 8 are determined by passing an additional plane, Q, through edge D of the prism, which is perpendicular to H and the apex of the pyramid (S).

This plane intersects the pyramid, creating section SKL, which intersects edge D at points 7 and 8. The visibility of the intersection line is determined by the visibility of the crossing edges and faces of the two polyhedra. Points 5 and 8 are hidden because they lie on the hidden face of either the prism or the pyramid: (.)5 Є DE and (.) 8 Є ABS.

- **Applications :**

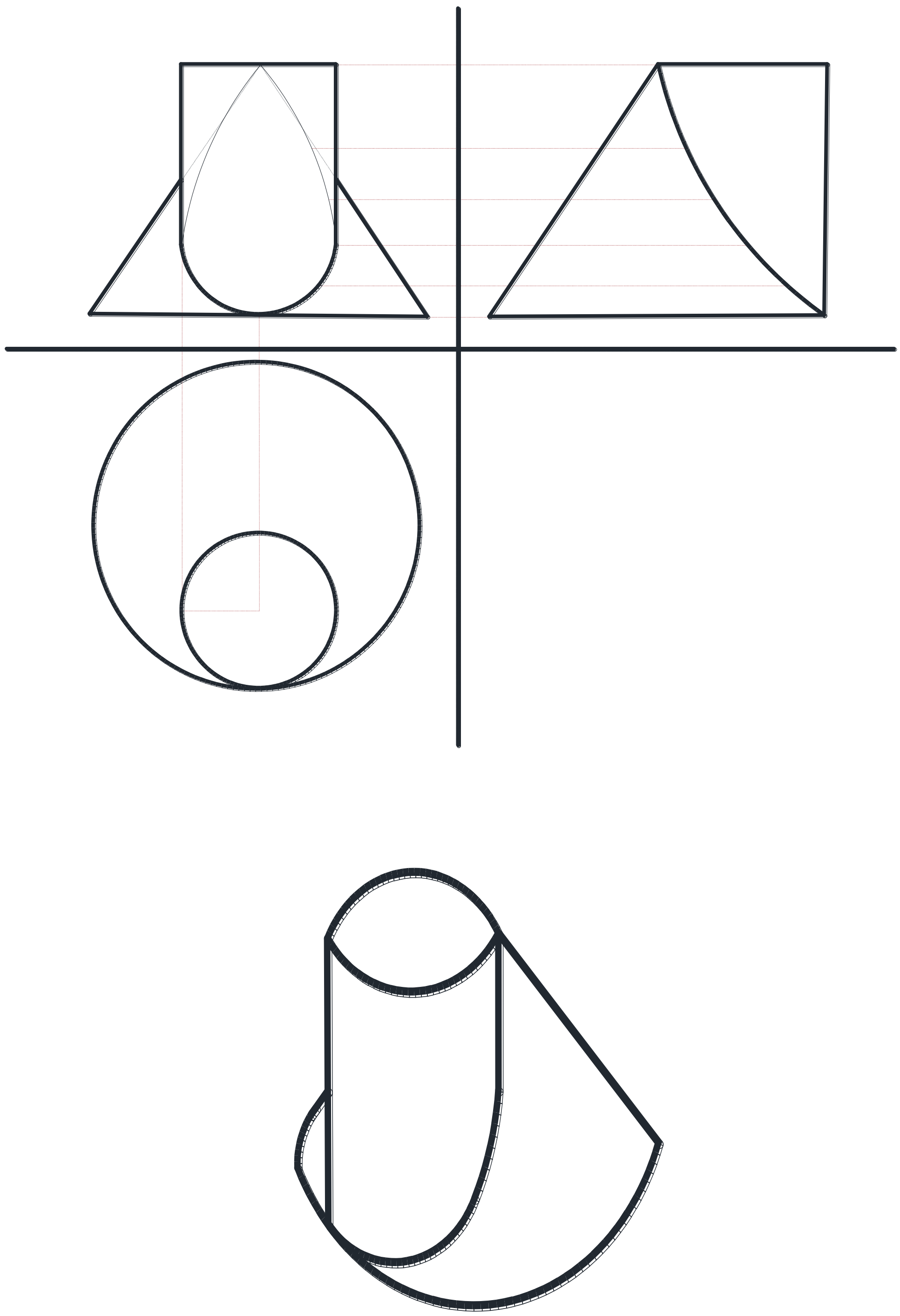


Figure 76: source author

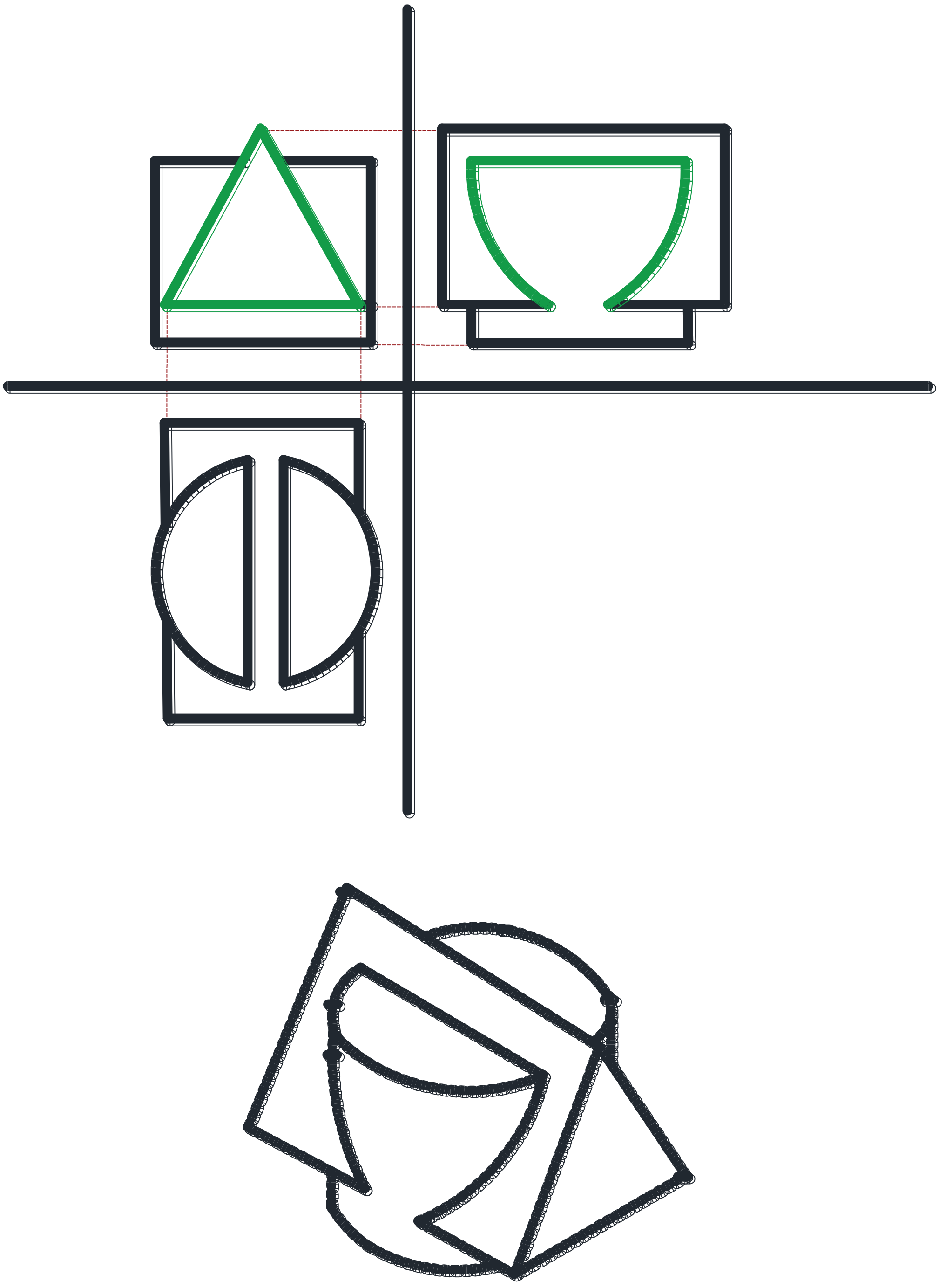


Figure 77: source author

Exercice :

-What is the method for finding the intersection between a polyhedron and a plane? Provide an example.

-How do we determine the points where a polyhedron and a line intersect? Provide an example.

-Outline the fundamental principles of intersecting two polyhedra.

COURSE N-09 : Introduction to Three-Dimensions Geometry and Axonometric projection

1 Understanding Three-Dimensional Space:

Comprehending three-dimensional space holds fundamental importance across various disciplines, including mathematics, physics, computer graphics, and architecture. Unlike two-dimensional space, which is defined by length and width alone, three-dimensional space introduces depth as the third dimension, providing a sense of volume and solidity.

Within three-dimensional space, objects possess height, width, and depth, facilitating a more precise depiction of the physical environment. This added dimension of depth plays a critical role in discerning distances and spatial connections between objects.

Depth perception denotes the capability to gauge the distance of objects from an observer, along with their relative distances from each other within space. It relie on visual cues such as relative proportions, overlapping, shading, texture gradients, and perspective. For instance, objects situated farther away appear smaller, while those closer seem larger in relation to the observer.

Spatial relationships in three-dimensional space encompass understanding how objects are positioned concerning each other within a given spatial context. This encompasses concepts like proximity, orientation, direction, and relative placement. Spatial relationships are indispensable for activities such as navigation, design, and spatial reasoning.

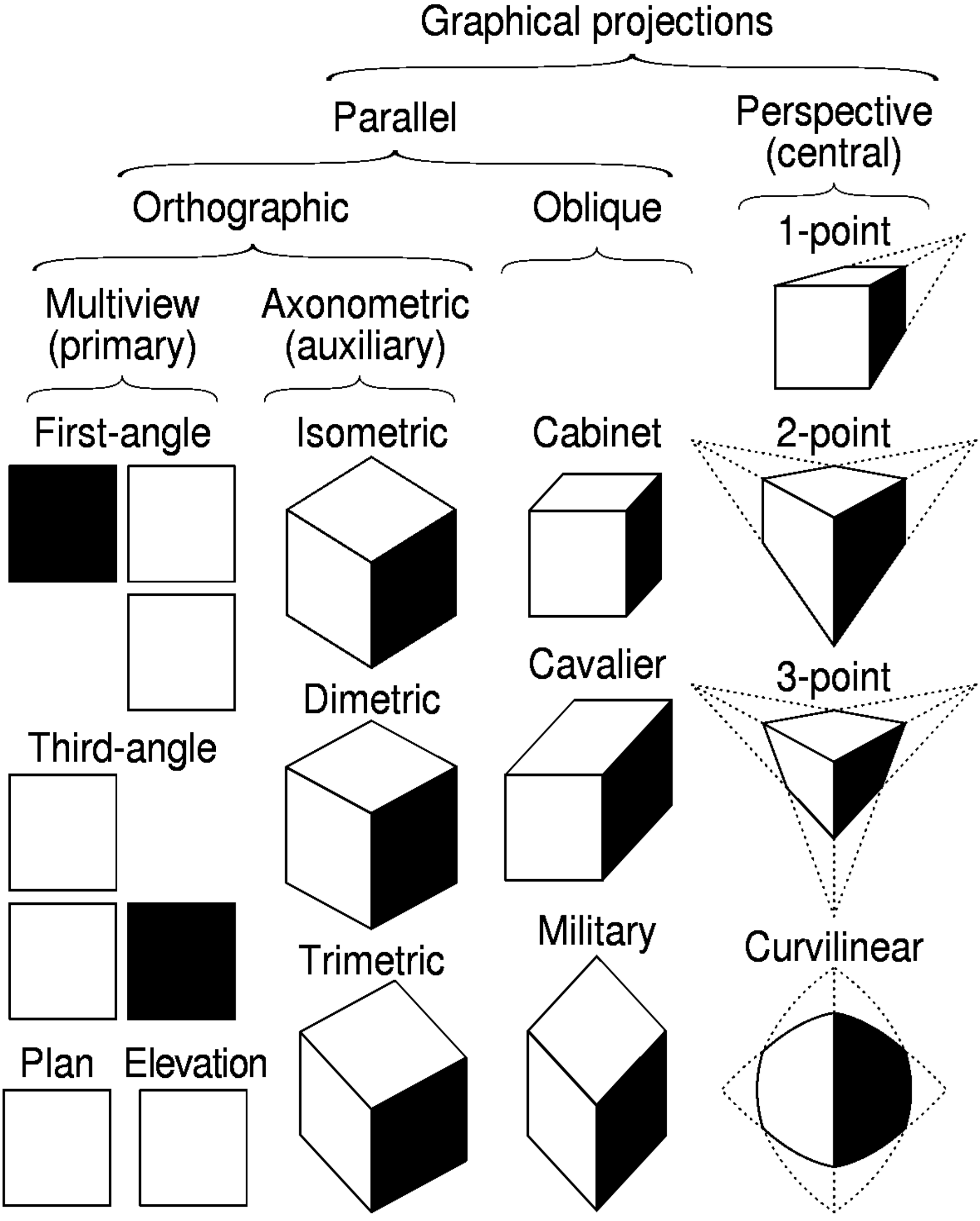


Figure 78: source: google image

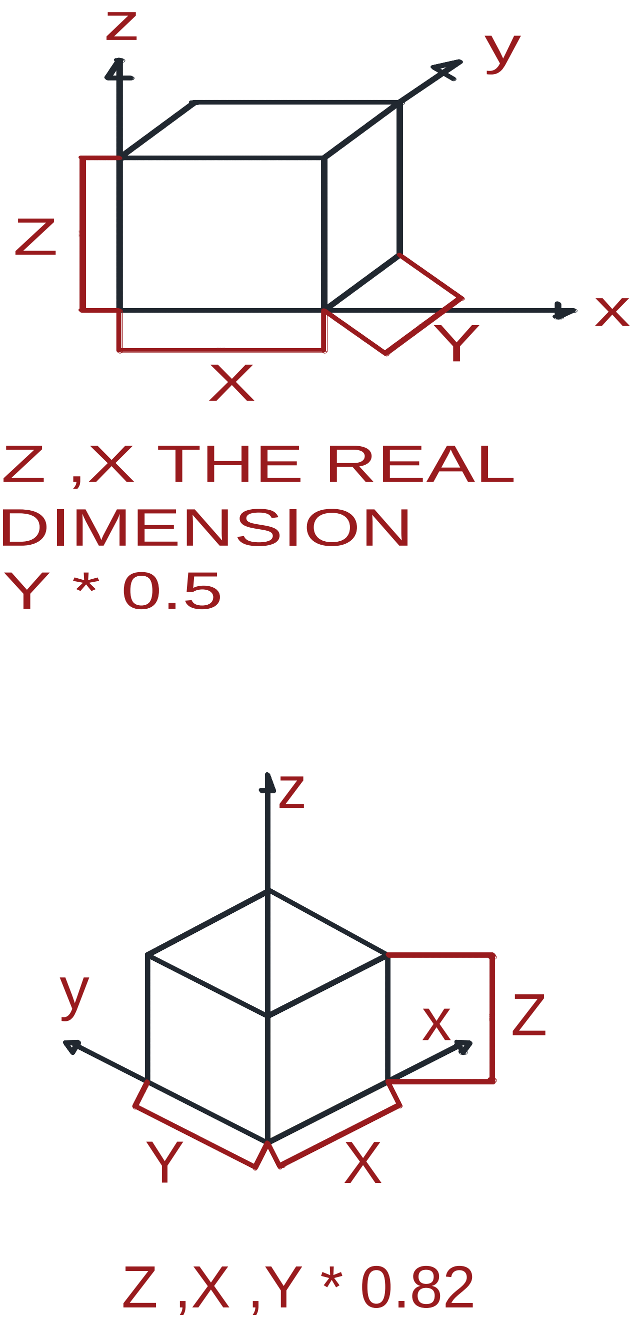


Figure 79: isometric and cavalier coefficient source: author

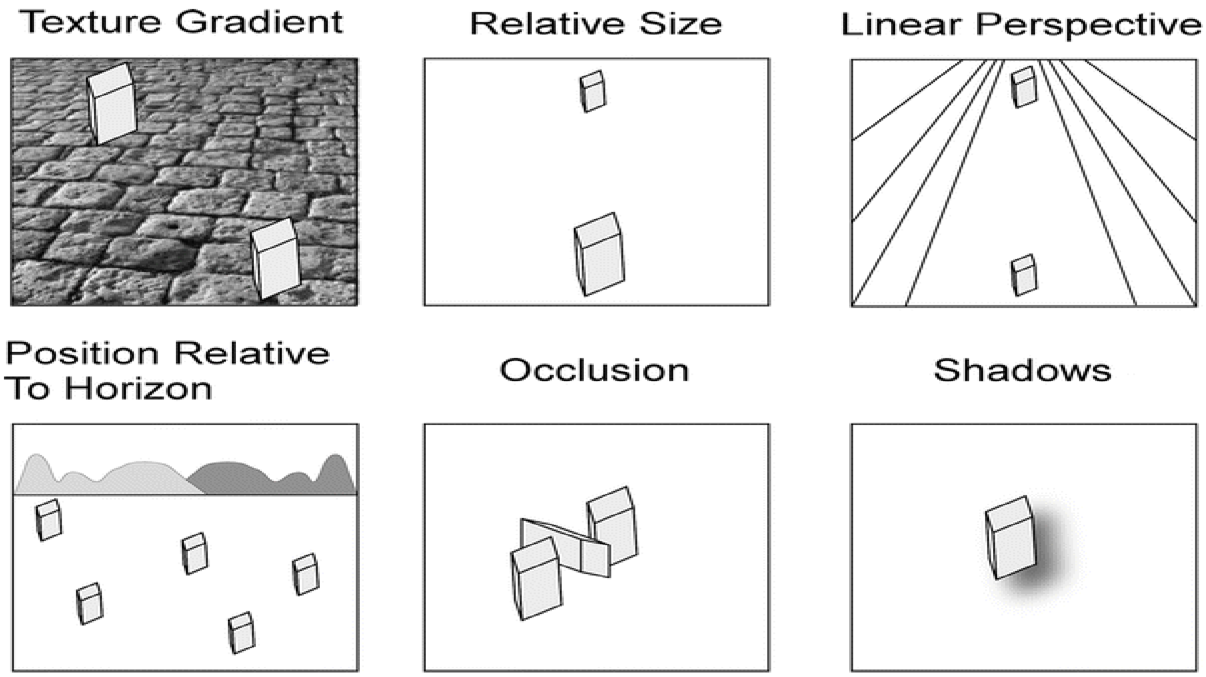


Figure 80; Exemples of Depth perception in three-dimensional space, source : https://link.springer.com/referenceworkentry/10.1007/978-3-319-16999-6_2758-1

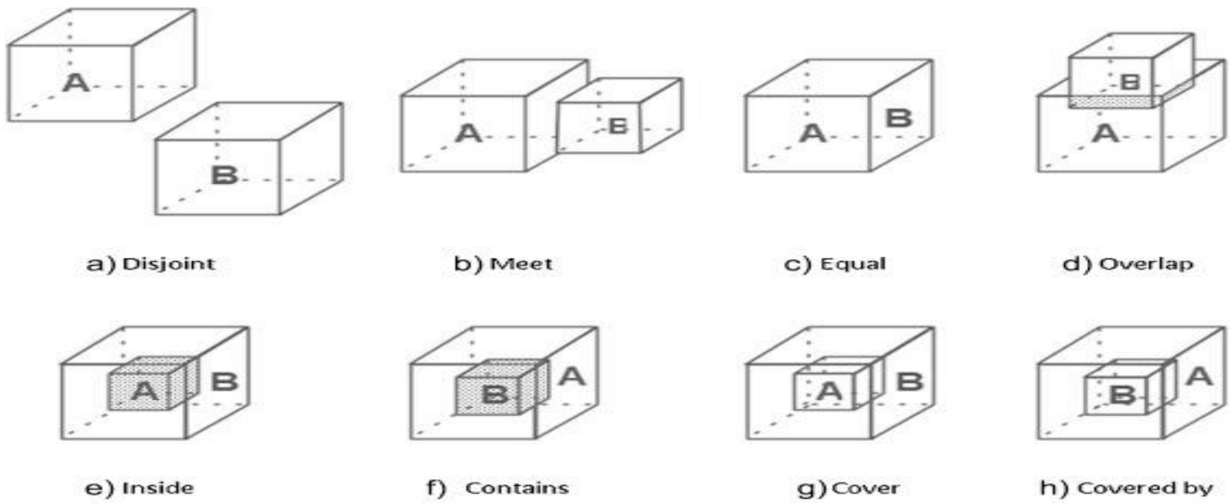


Figure 81; Exemples of Spatial relationships in three-dimensional space, source:

https://www.sciencedirect.com/science/article/pii/S0166361516300598?__cf_chl_rt_tk=4S2Sz3U6cFYymKG.u1.Us.D9KZHAmEj6ohV8yBY8Txc-1715126711-0.0.1.1-1386

In essence, the concept of three-dimensional space introduces depth as a pivotal dimension, enabling a richer comprehension of the physical world. Depth perception enables the accurate assessment of distances and spatial arrangements, while spatial relationships elucidate how objects are situated relative to one another within three-dimensional space.

3 Demonstrating how to represent three-dimensional objects on a two-dimensional plane using projection techniques.

Exploring projection techniques entails the process of depicting three-dimensional objects on a flat surface. This is accomplished through methods like orthographic, perspective, or axonometric projection.

Orthographic projection, widely used in technical fields, projects the object onto the plane along parallel lines, resulting in a two-dimensional representation where all lines are perpendicular. It accurately preserves the object's shape and size but lacks depth perception.

Perspective projection mimics human vision by projecting the object onto the plane from a specific viewpoint, causing lines to converge towards a vanishing point. This creates a sense of depth, making distant objects appear smaller. It's commonly employed in art and architecture for realism.

Axonometric projection : (as illustrat in figure) a form of parallel projection, projects the object onto the plane using non-parallel lines, resulting in a visually pleasing but distorted representation. It maintains relative proportions and is favored in technical drawing and design.

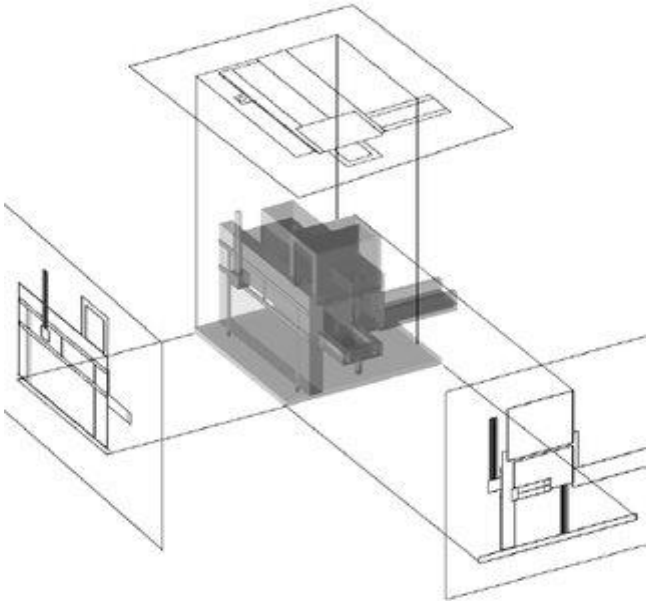


Figure 82: Top View and side views source: https://www.alamy.com/axonometry-three-dimensional-drawing-of-alfresco-bar-aka-dehors-image366086647.html

- **Applications :**

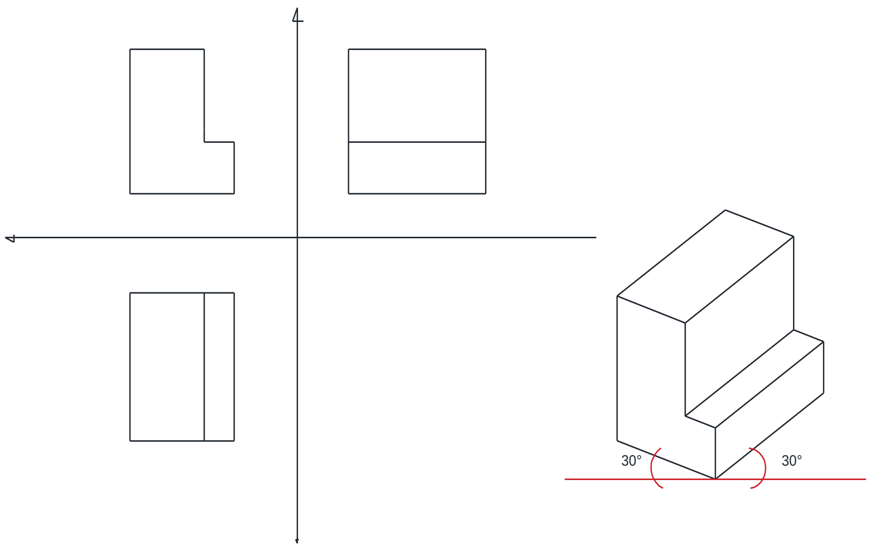


Figure 83: Top View , side views and axomometry source author

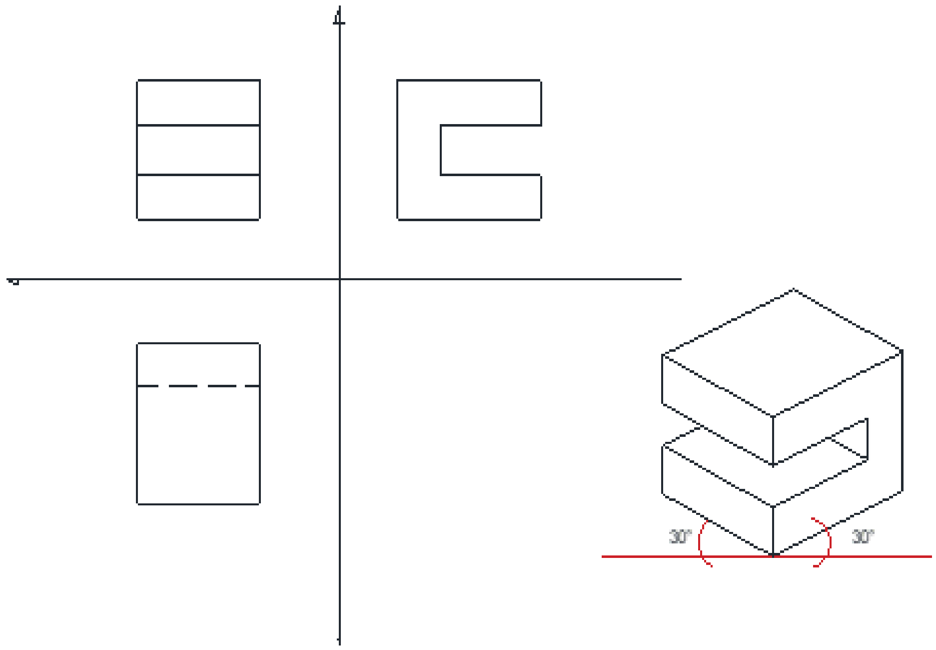


Figure 84: Top View , side views and axomometry source author

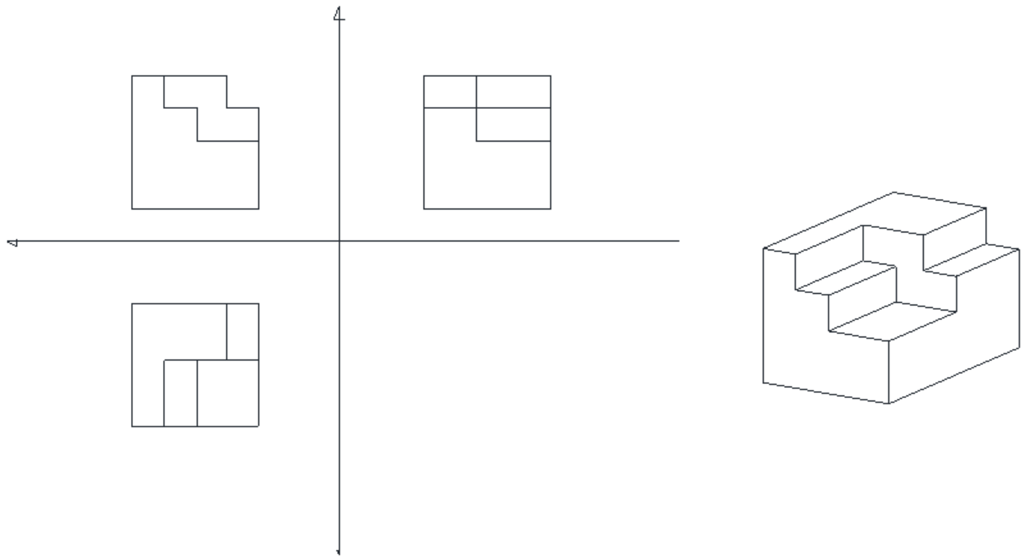


Figure 85: Top View , side views and axomometry source author

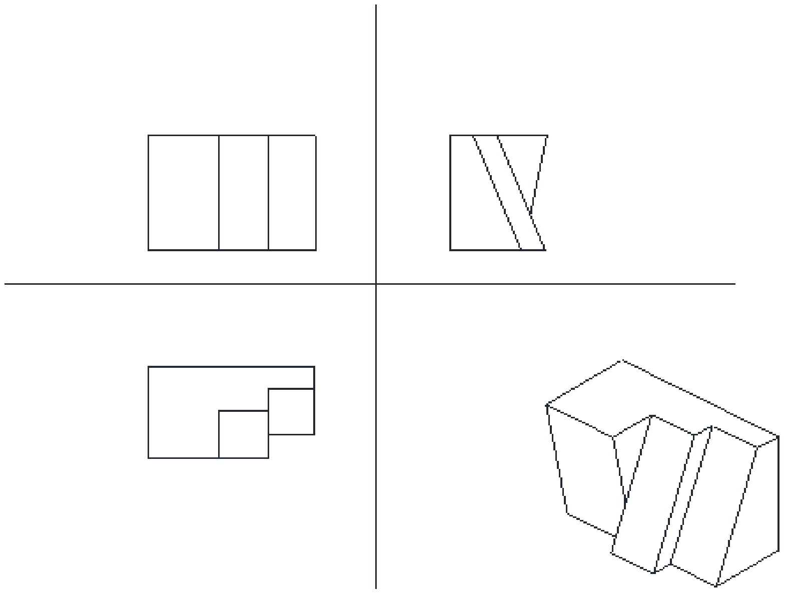


Figure 86: Top View , side views and axomometry source author

In essence, the choice of projection method depends on the task's requirements, each offering distinct advantages and limitations. These techniques find application in diverse fields like engineering, architecture, art, and design, enabling a wide array of visual representations.

4 solid modeling techniques for representing complex 3D objects :

Solid modeling techniques are indispensable for accurately representing complex three-dimensional objects across diverse fields like engineering, architecture, manufacturing, and computer graphics. Unlike surface or wireframe modeling that focuses on external surfaces, solid modeling involves creating closed volumes that faithfully depict an object's physical properties and geometry.

These techniques empower designers and engineers to generate detailed and lifelike representations of intricate objects, ranging from components to entire structures. They offer precise control over shape, size, and features, facilitating design analysis, visualization, and simulation.

-Several solid modeling approaches exist:

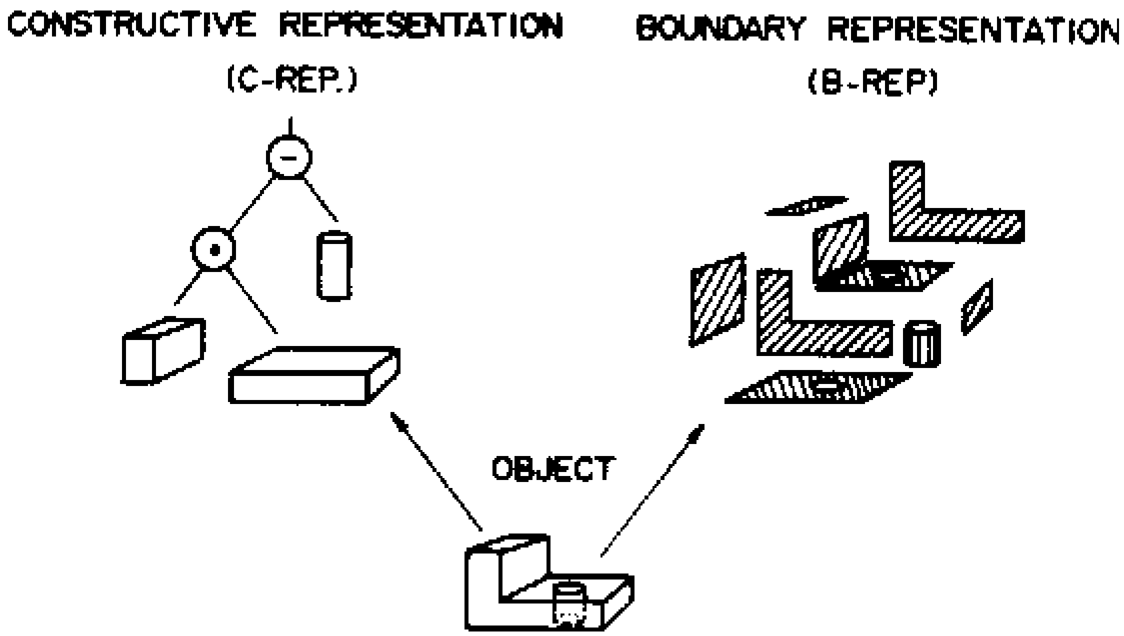
a-Boundary Representation (B-Rep): B-Rep depicts solid objects as collections of bounded surfaces, each defined by geometric properties such as curvature and connectivity. Widely used in CAD software, B-Rep offers flexibility for representing complex shapes.

b-Constructive Solid Geometry (CSG): CSG constructs intricate shapes by combining simple geometric primitives (e.g., cubes, cylinders) through Boolean operations. This method allows for composing complex shapes from basic building blocks.

c-Parametric Modeling: Parametric modeling defines object geometry using mathematical parameters, enabling flexible and easily modifiable models. Changes to one parameter automatically update the entire model, making it ideal for iterative design and product development.

d-Feature-Based Modeling: This approach defines specific object features (e.g., holes, fillets) that are combined to create the final solid model. It offers a structured method for creating objects with repetitive features.

In summary, solid modeling techniques provide powerful tools for accurately representing complex three-dimensional objects. By leveraging methods like boundary representation, constructive solid geometry, parametric modeling, and feature-based modeling, designers and engineers can create detailed and realistic models that streamline design, analysis, and manufacturing processes.



-

Figure 87: source: google image

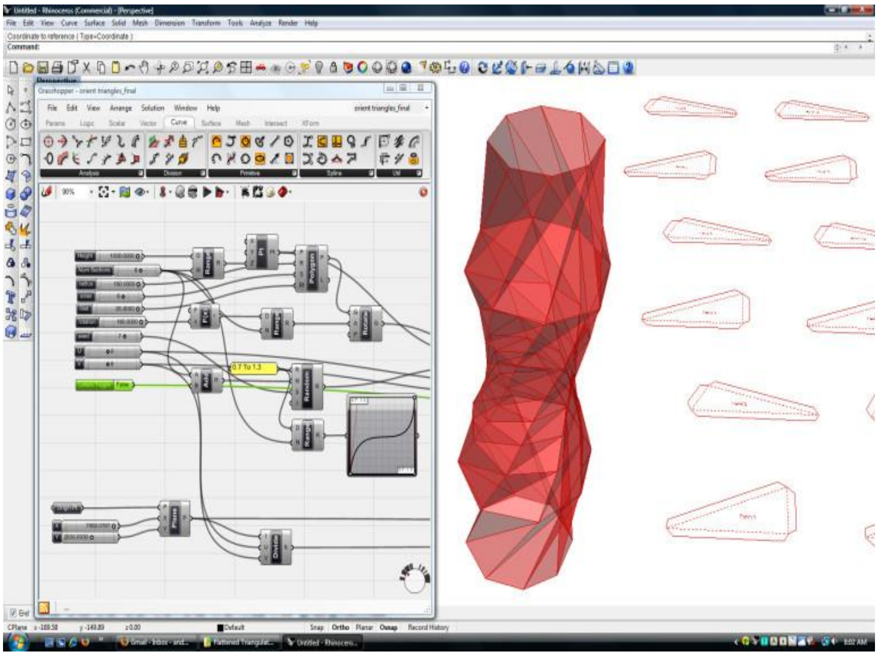


Figure 88: Parametric Modeling source: google image

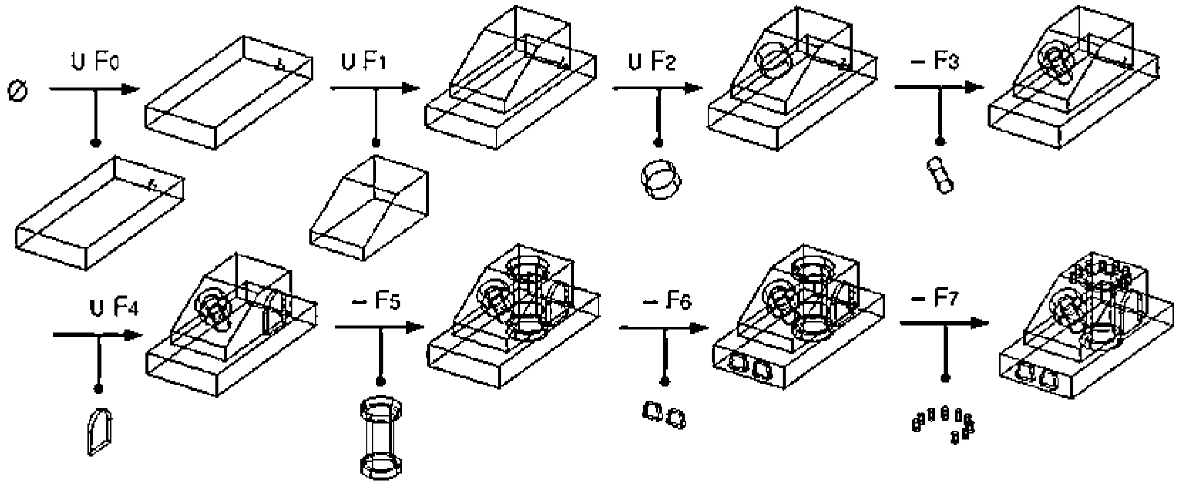


Figure 89 : Feature-Based Modeling, source : google image

- **Applications :**

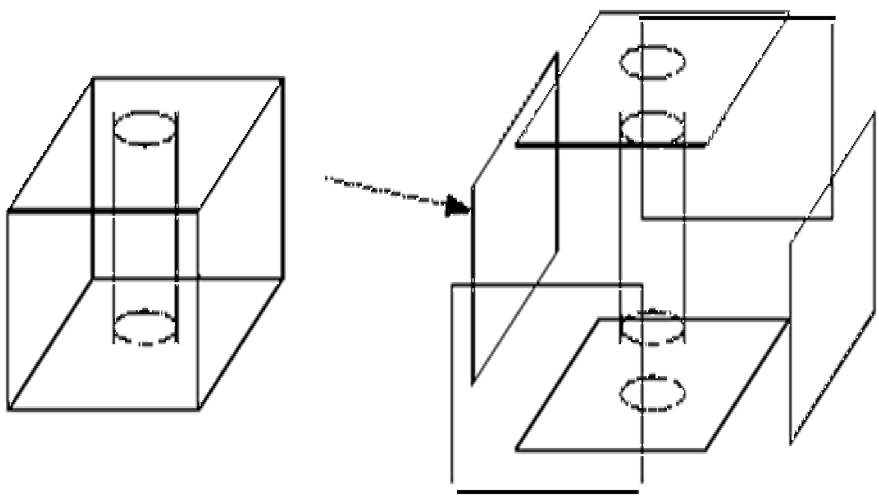


Figure 90 : Boundary Representation (B-Rep) : source author

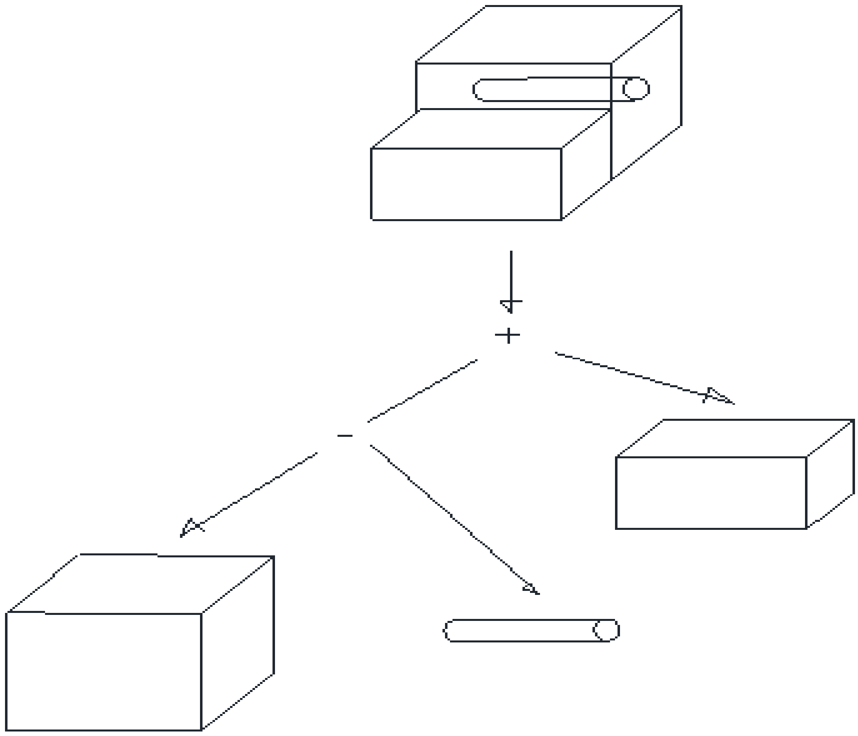


Figure 91: Constructive Solid Geometry (CSG): source author

Exercice:

- Define and explain the concept of axonometry in architectural representation.
- Discuss the advantages and disadvantages of using axonometric drawings compared to other forms of architectural representation, such as perspective drawings.
- Describe the different types of axonometric projections (isometric, dimetric, trimetric) and when each type is typically used in architectural design
- In your CAD software, you create a design by adding individual geometric features such as circles, lines, and curves. Later, you modify the design by

directly editing these features. What type of modeling technique are you using?a) Parametric Modeling

COURSE N°10: Generalities about perspective

1 Introduction:

Perspective is the art of depicting 3D objects on a 2D surface to simulate depth perception, resembling how the human eye perceives depth. Achieved through converging lines and vanishing points, it's crucial in art and architecture for lifelike depictions. Linear perspective converges parallel lines to a vanishing point, while atmospheric perspective uses color and tone for depth illusion.

This concept is fundamental not only in visual arts but also in engineering, design, and photography, aiding spatial representation for effective communication. Additionally, conic projection, also called perspective projection, projects 3D objects onto a 2D surface by imagining them within a cone of vision. It distorts objects, making closer points appear larger, akin to natural vision's depth perception. Widely used in cartography and technical drawing, conic projection facilitates the creation of maps and illustrations while preserving some aspects of objects' 3D structure[2].

[2] **Note that this course will be developed in the second semester, dealing with the variation of the distance from the viewpoint, angle values, perspective using the method of actual and reduced distance points, the development of a helical ribbon, the trapezoidal grid ,… »**

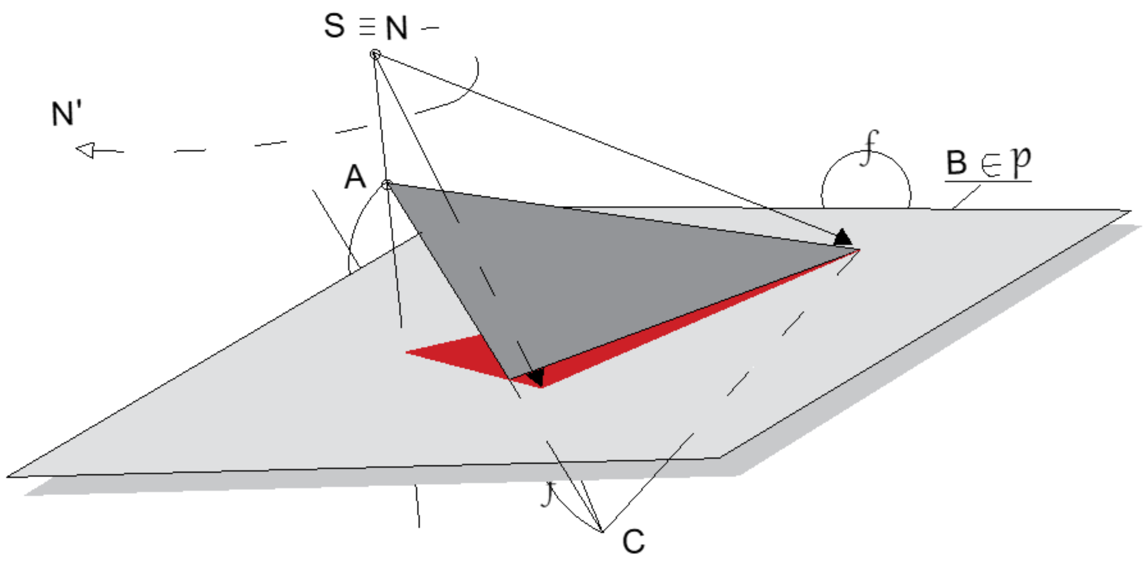


Figure92: source author

Every perspective, from a geometric standpoint, involves projecting points from space onto a designated plane using a defined projection center and plane. This projection is represented by a transformation function, f(P, S), where each point A in space corresponds to its image A' on the plane. The image A' is determined by the intersection of the line connecting the projection center S and point A (SA) with the projection plane P. In essence, perspective transforms spatial points into their respective images on a plane through a defined geometric process.

-Types of plane and projections :

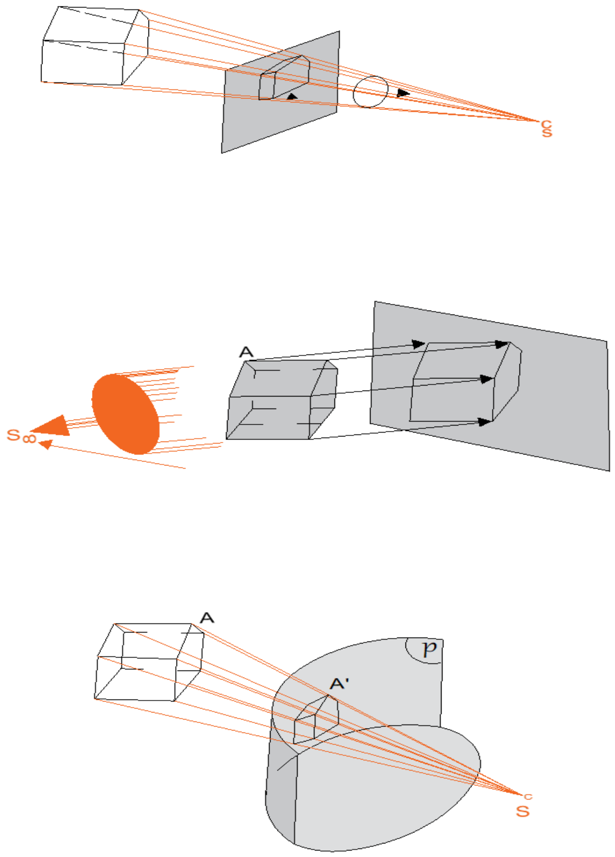


Figure 93: source author

2-**Principles of Perspective Drawing**

Perspective drawing employs several fundamental principles essential for creating depth and realism in artworks:

Horizon Line: This horizontal line denotes the viewer's eye level and helps determine the positioning of objects in a drawing.

Vanishing Points: Points on the horizon line where parallel lines converge, aiding in establishing spatial relationships and object direction.

Vanishing Lines: Imaginary lines drawn from object edges to vanishing points, assisting artists in achieving depth perception.

The historical significance of perspective drawing is profound:

Renaissance Era: Pioneered during this period, linear perspective transformed art, spearheaded by figures like Brunelleschi and Alberti.

Artistic Innovations: Perspective enabled artists to create lifelike works, advancing painting, architecture, and sculpture.

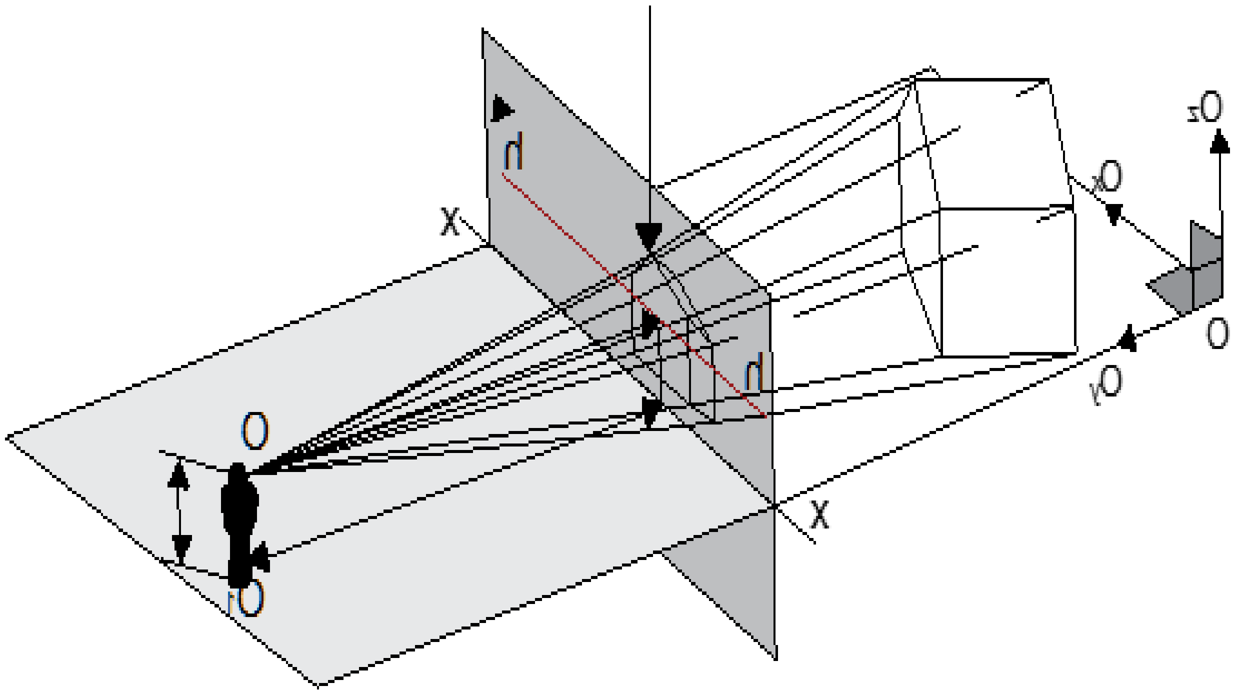


Figure 94: source author

- **A system of perspective :**

The Perspective system, like the Double Orthogonal Projection (DOP) system, consists of two main elements: the Geometrical Plane (G) and the Picture Plane (T), which intersect along the Ground Line (xx). These planes are perpendicular to each other.

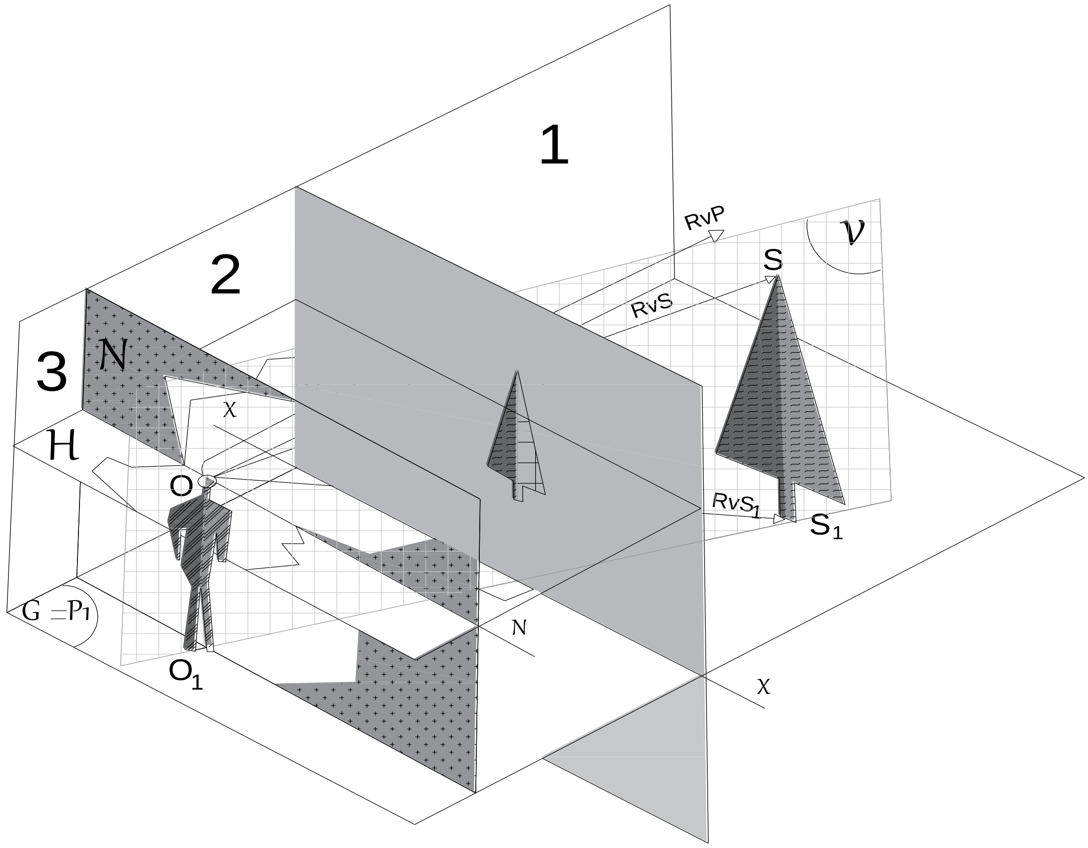


Figure 95 : source author

The observer's eye (O) and its projection (O1) on the Geometrical Plane are positioned at a Principal Distance (d) from the Picture Plane. The main visual ray (OP) is perpendicular to the Picture Plane at the Principal Vanishing Point (P). The eye (O) is at a Principal Height (h) above the Geometrical Plane.

The Horizon Line (hh) passes through the Principal Vanishing Point and is parallel to the Ground Line. The distance between these lines equals the Principal Height.

Objects, like a tree with its top (S) and base (S1), are typically behind the Picture Plane. The visual rays from these points intersect the Picture Plane at points S' and S'1, representing their perspectives.

The Frontal Plane (N) through the observer's eye is parallel to the Picture Plane, dividing the space into three zones:

Zone of the object
Observer's zone
Virtual zone, where perspectives are inverted.
Objects on the neutral plane have perspectives at infinity. The Horizontal Plane (H) through the observer's eye intersects the Picture Plane at the Horizon Line, meaning any object's perspective on this plane lies on the Horizon Line.

3-Perspectives in Geometry: Artistic and Scientific

3-1 Artistic Perspective:

<u>Linear Perspective:</u> Utilizes vanishing points and lines to give the illusion of depth and space in 2D artworks.

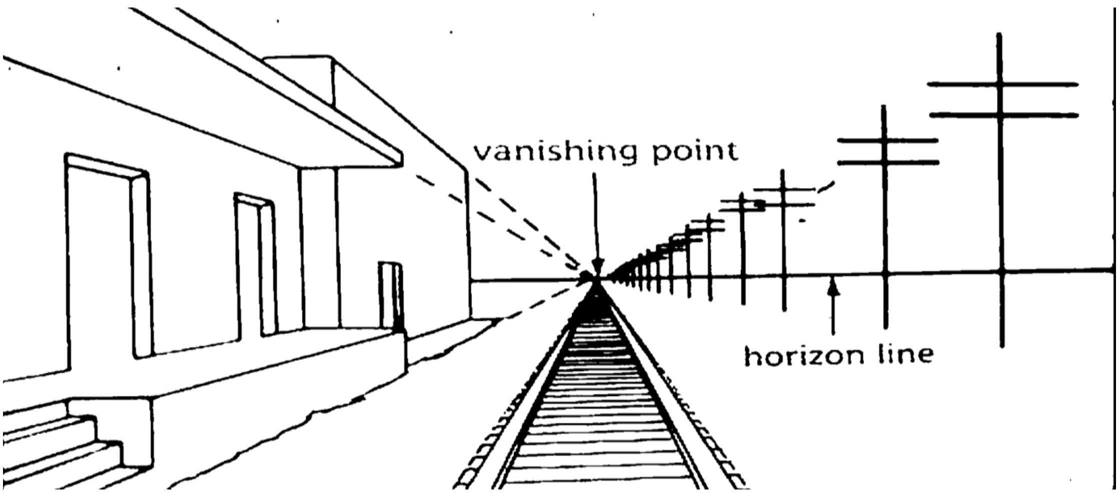


Figure 96: source: google image

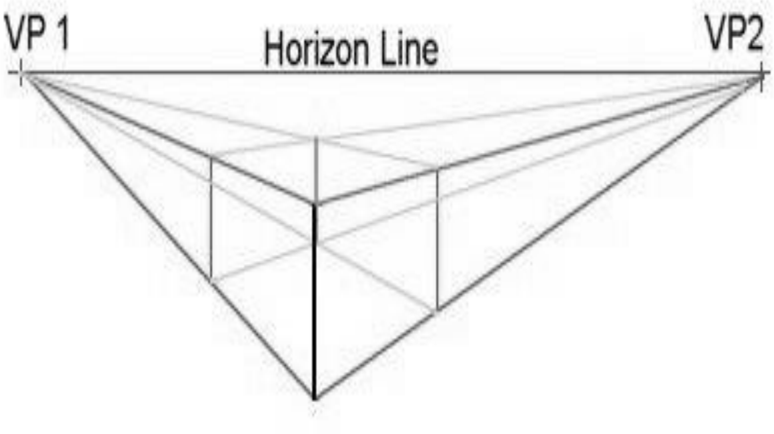


Figure 97: source: google image

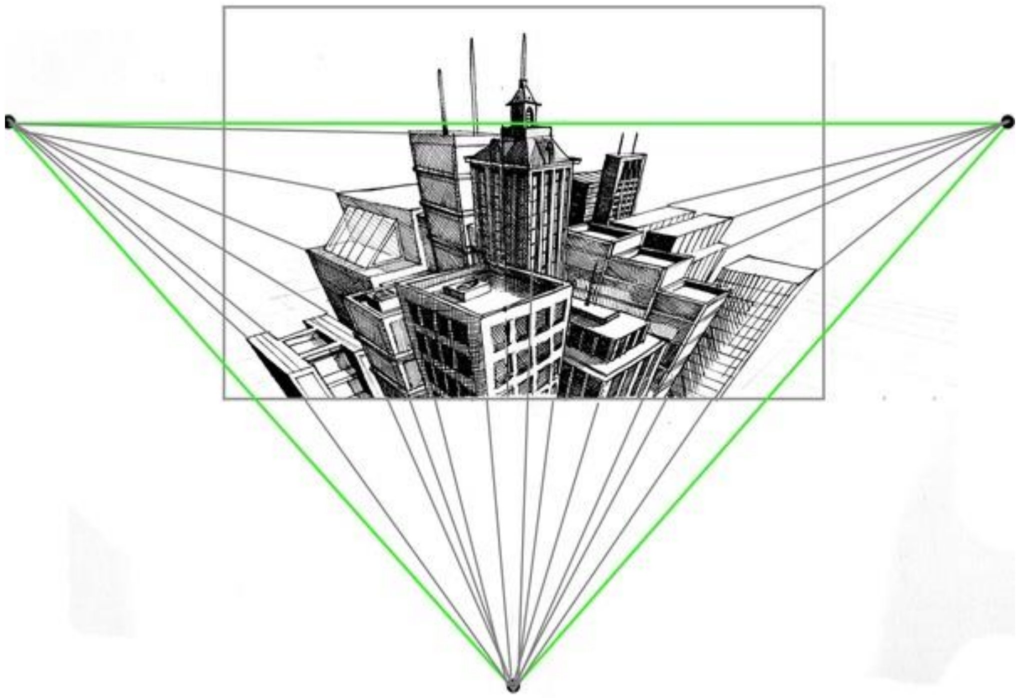


Figure 98: source: google image

- *Aerial Perspective*: Also called atmospheric perspective, it uses color and shading to convey distance and depth by simulating atmospheric effects.

Figure 99: source: google image

- *Foreshortening*: Adjusts object proportions to create depth and volume in 2D, commonly seen in figure drawing and portraiture.

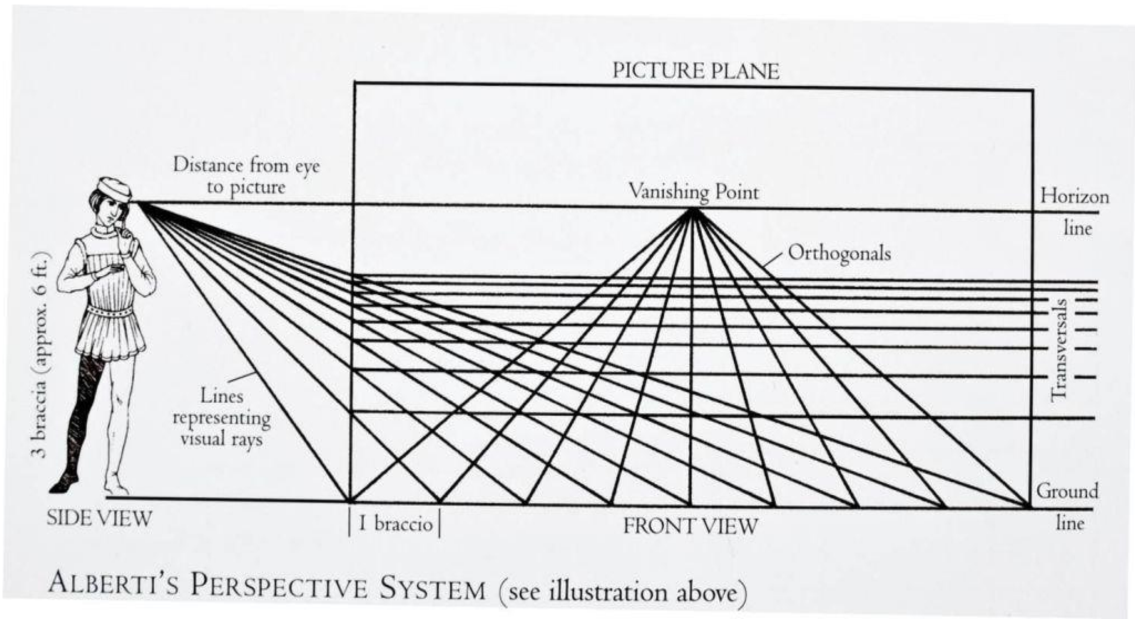


Figure 100:Google image

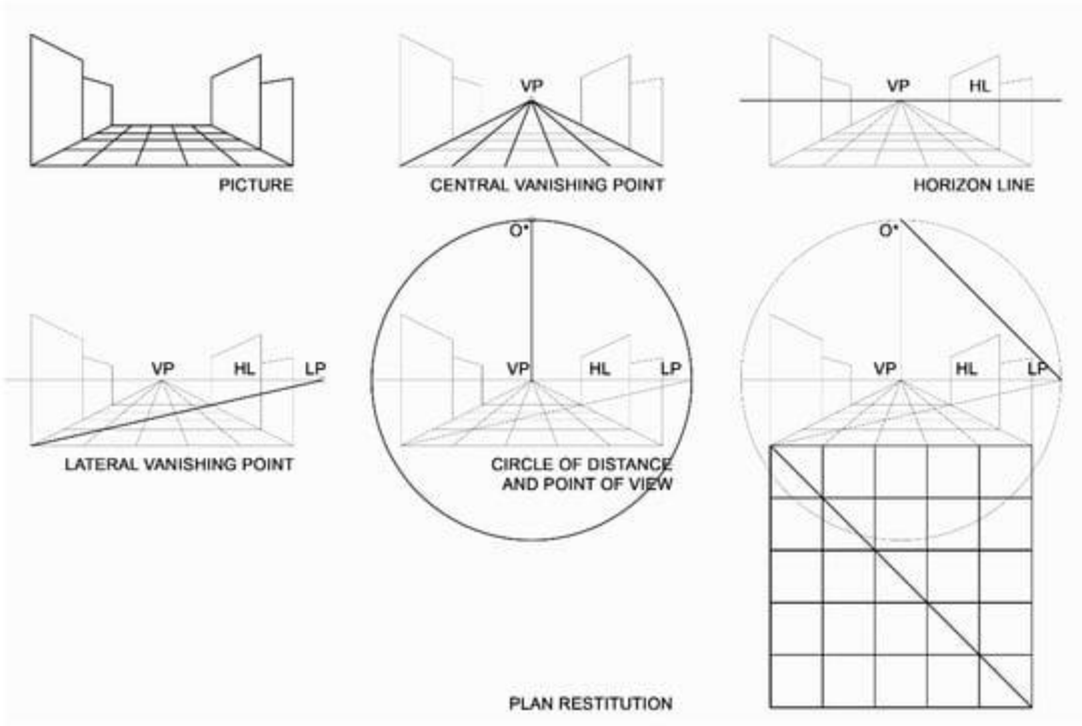


Figure 101: Google image

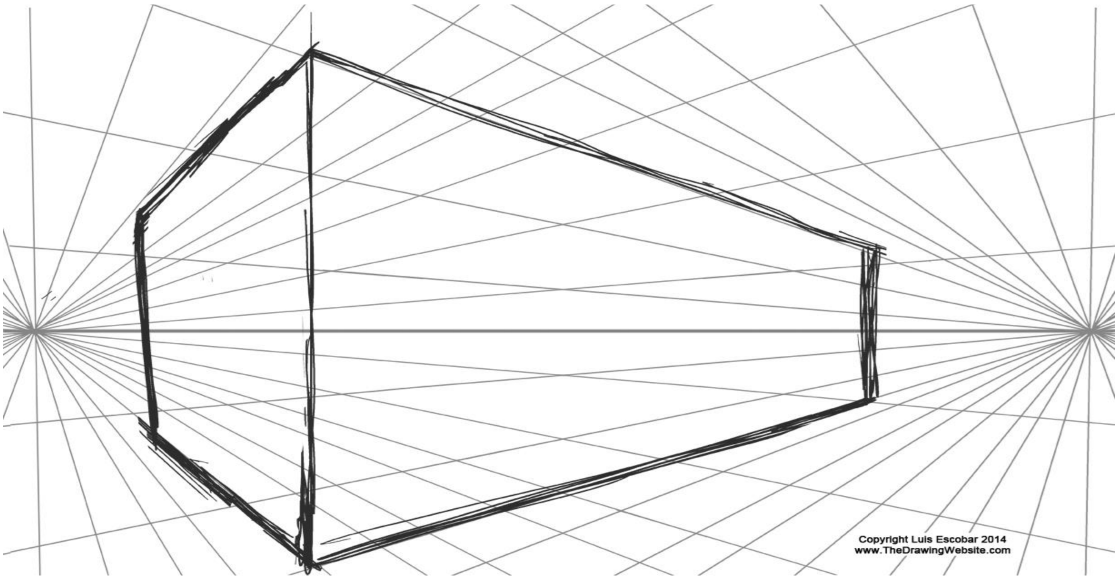


Figure 102: Google image

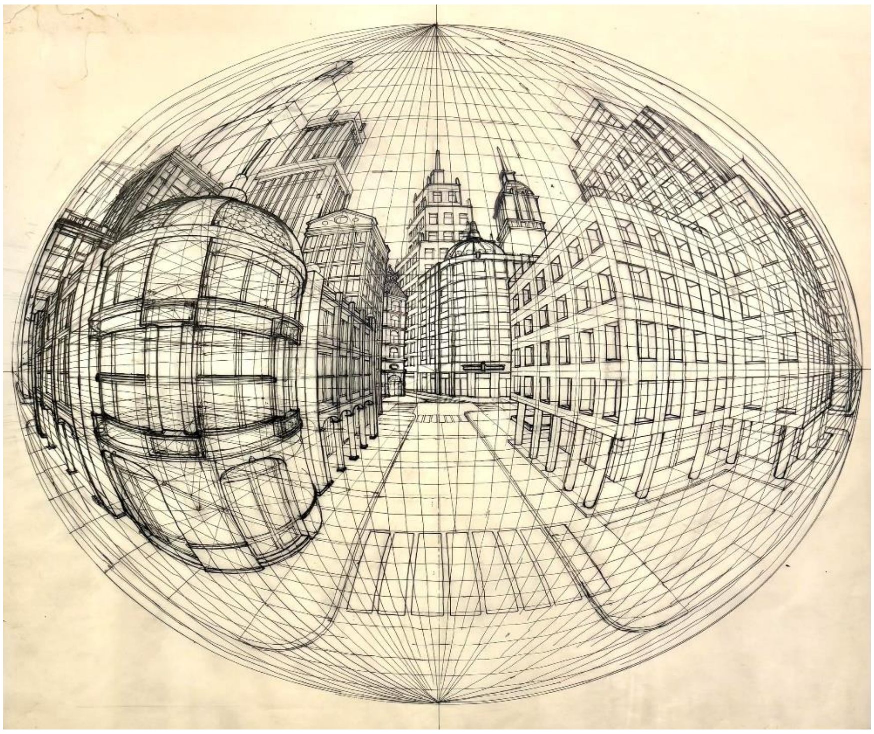


Figure 103: source : by rafael araujo

https://www.facebook.com/rafaelaraujoart

3-2 Scientific Perspective:

- **<u>Analytical Geometry:</u>** Applies algebraic and geometric methods to analyze shapes, often used in mathematical modeling and computer graphics.

  Analytical geometry, also **known as coordinate geometry**, applies algebraic and geometric methods to analyze shapes, often used in mathematical modeling. Here are the steps to perform a perspective analytical geometry analysis:

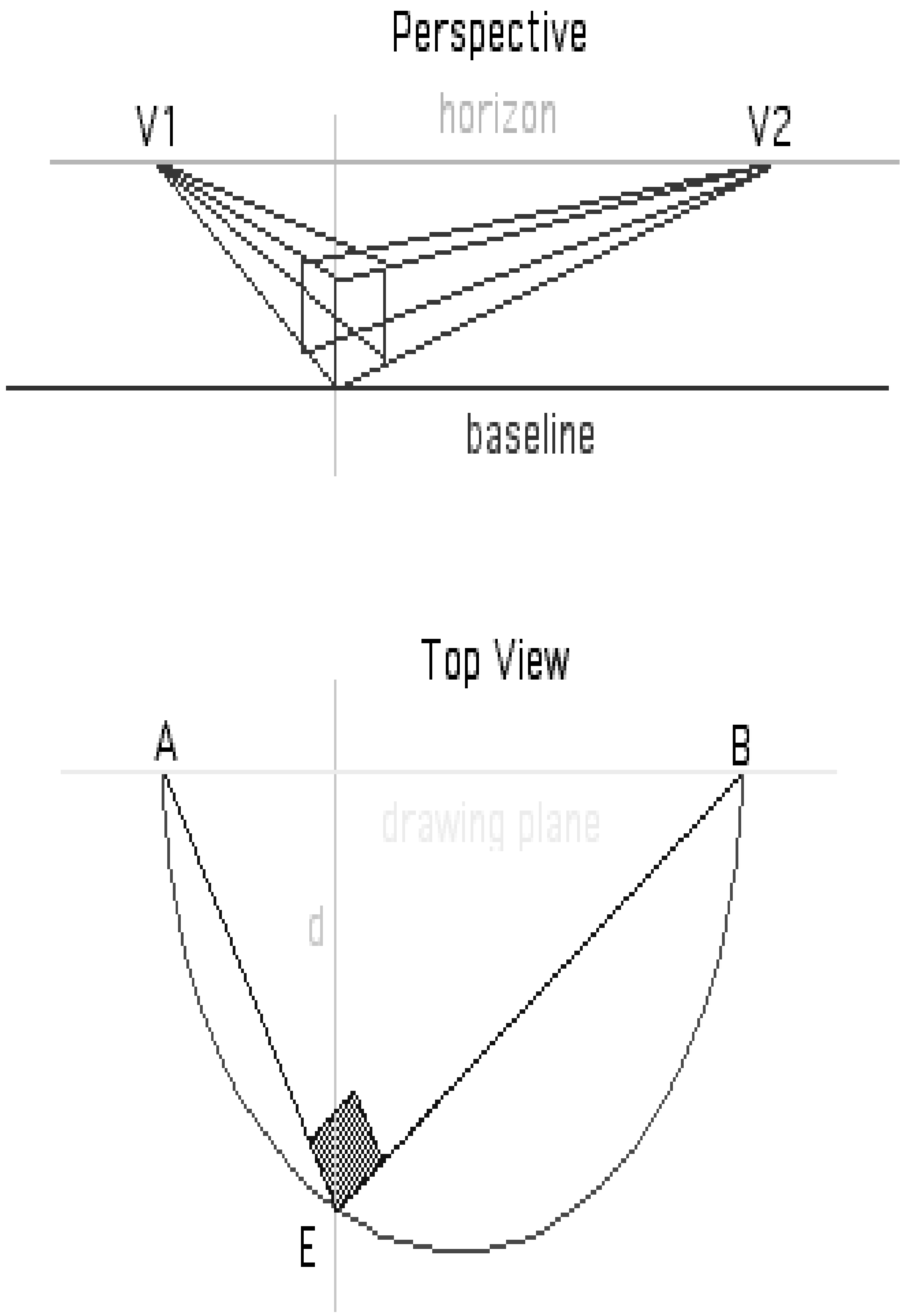


Figure 104 : source author

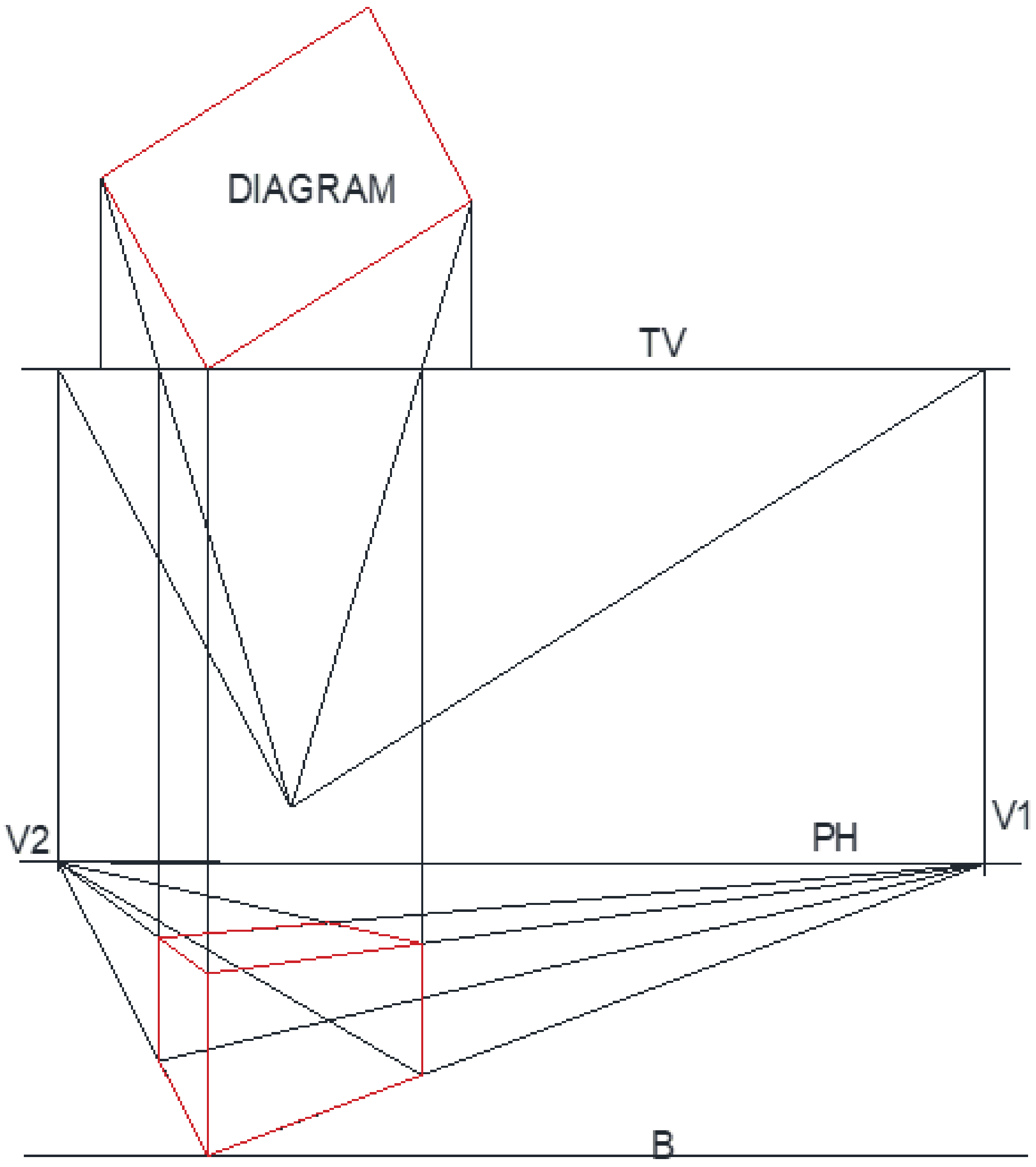


Figure105 : source author

The Architects' Method, also known as Figure 95 , involves initially gathering all necessary data, including the length, depth, width, and height of the object to be depicted, and arranging it on a drawing sheet with meticulous care. This includes creating a plan view and an elevation view, presenting a comprehensive outline of the object, with the plan view positioned on the left side to align with the width of the sheet, and the elevation view centered vertically to match the height of the sheet. Additionally, the ground line is extended across the entire width of the drawing sheet to provide a reference point for the representation

- ***Projective Geometry*:** Studies geometric figures under projection, exploring concepts such as duality and projective transformations.

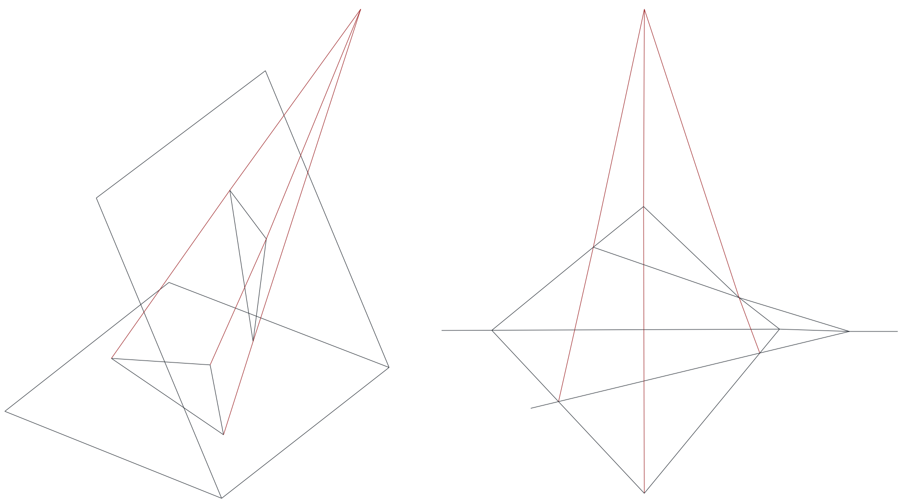


Figure 106 : source author

- **Differential Geometry:** Investigates the curvature and topology of surfaces and curves, with applications in physics, engineering, and computer science

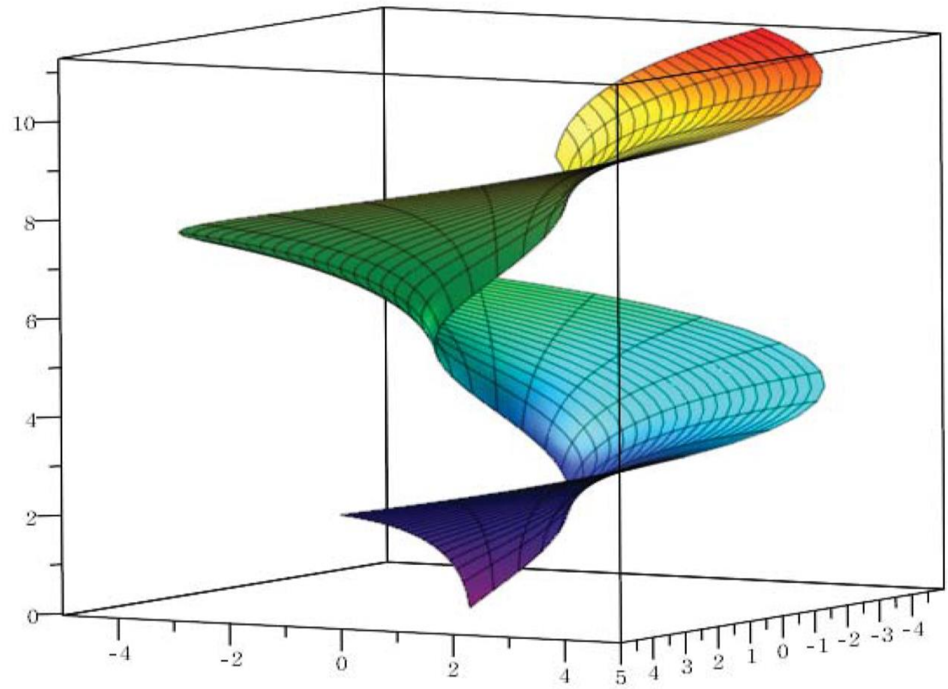


Figure 107: google image

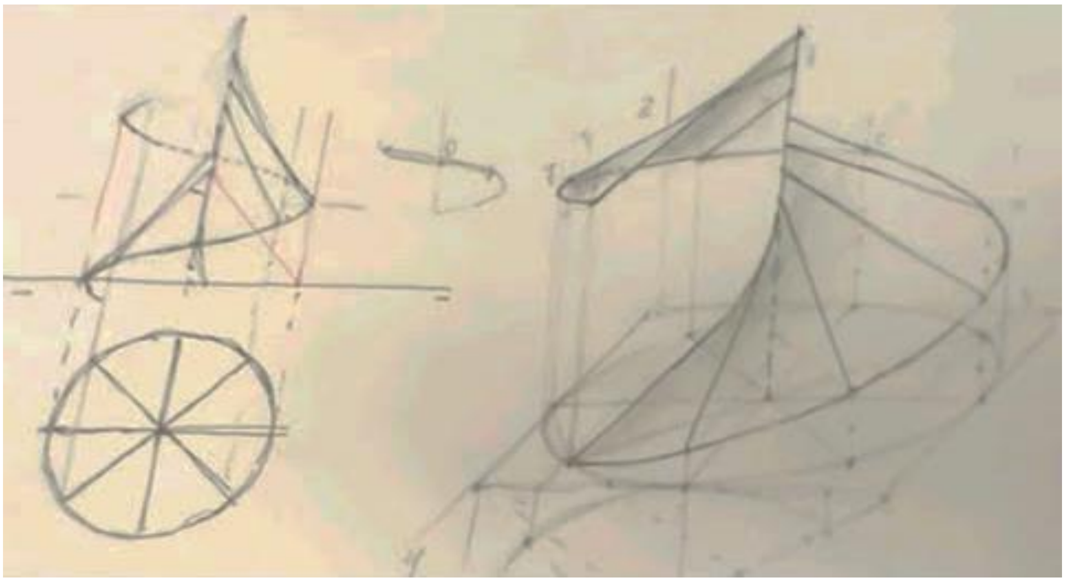


Figure 108 : google image

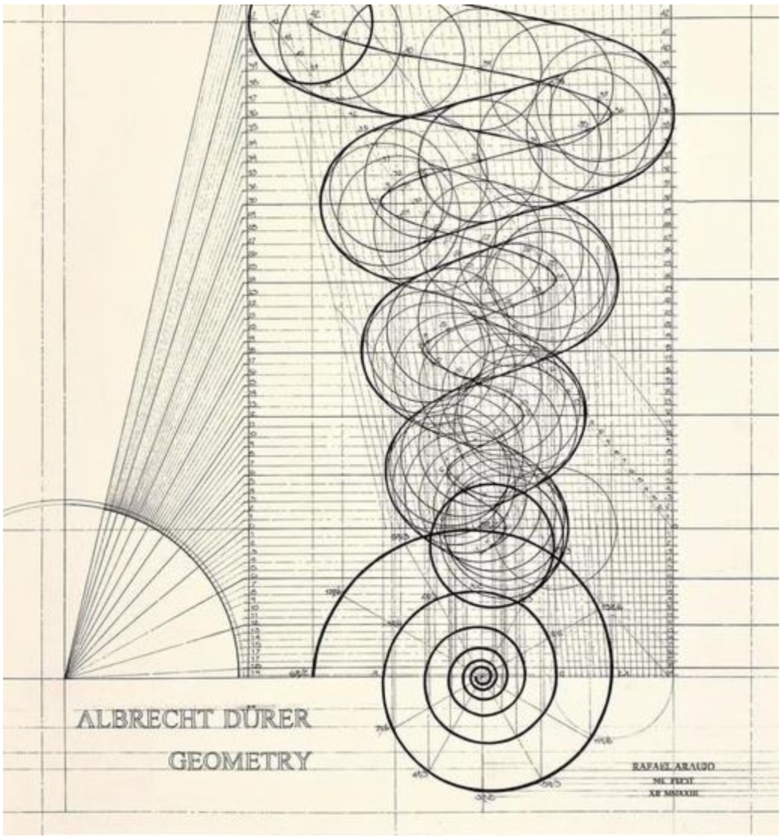


Figure 109 : source : by rafael araujo

https://www.facebook.com/rafaelaraujoart

Exercice :

Instructions:

1. Read each definition carefully.
2. Choose the correct term that matches the given definition.
3. Write the corresponding letter of the correct term in the space provided.

Questions:

1. Which term refers to the point where parallel lines appear to converge in a perspective drawing?
   - ☐ A. Vanishing Point
   - ☐ B. Horizon Line

- ☐ C. Foreshortening
- ☐ D. Orthogonal Lines

2. What is the line representing the eye level of the observer in a perspective drawing called?
   - ☐ A. Vanishing Point
   - ☐ B. Horizon Line
   - ☐ C. Foreshortening
   - ☐ D. Orthogonal Lines
3. Which term describes the technique used to represent objects as they appear shorter or distorted due to their distance from the viewer?
   - ☐ A. Vanishing Point
   - ☐ B. Horizon Line
   - ☐ C. Foreshortening
   - ☐ D. Orthogonal Lines
4. What are the lines in a perspective drawing that converge towards the vanishing point called?
   - ☐ A. Vanishing Point
   - ☐ B. Horizon Line
   - ☐ C. Foreshortening
   - ☐ D. Orthogonal Lines
5. Which term refers to lines that are perpendicular to the picture plane in a perspective drawing?
   - ☐ A. Vanishing Point
   - ☐ B. Horizon Line
   - ☐ C. Foreshortening

- ☐ D. Orthogonal Lines

COURSE N=11: Application of the theory of shadows in monge projection, perspective and axonometry

1 Introduction :

Shadows, the absence of light caused by an obstruction, hold profound importance in the study of geometry, offering insights into spatial relationships, perspective, and the nature of light itself. In this course, we will explore how shadows are formed, their characteristics, and their impact on geometric shapes and structures. By understanding the principles of shadow projection and interaction, we gain a deeper appreciation for the geometric properties of objects and their representations. Moreover, we will delve into how shadows are utilized in various fields, including art, architecture, and engineering, to enhance visual communication and spatial comprehension.

2 Definition of Shadows :

In descriptive geometry, shadows are formed when an opaque object obstructs the path of light. This obstruction creates a region of darkness on a surface where the light is **blocked.**

3 Laws of light and shadow projection :

The projection of shadows follows several fundamental laws based on the interactions between light sources and objects. Here are the key principles:

- **Light Source and Object Relationship:**

Point Source: Produces sharp, well-defined shadows. The shadow size and shape depend directly on the object's distance from the light source.

Parallel Rays (Sunlight): Create consistent shadow sizes regardless of the object's distance, often used for outdoor scenes.

Diffuse Light Source: Results in softer, blurred shadows due to the scattering of light in multiple directions.

- **Shadow Projection on Different Planes:**

Horizontal Plane: Shadows on the ground or horizontal surfaces change length based on the light's angle.

Vertical Plane: Shadows cast on walls or vertical surfaces vary in shape and orientation depending on the light's position relative to the object and the surface.

Inclined Plane: Shadows on sloped surfaces are more complex, requiring precise calculations for accurate representation.

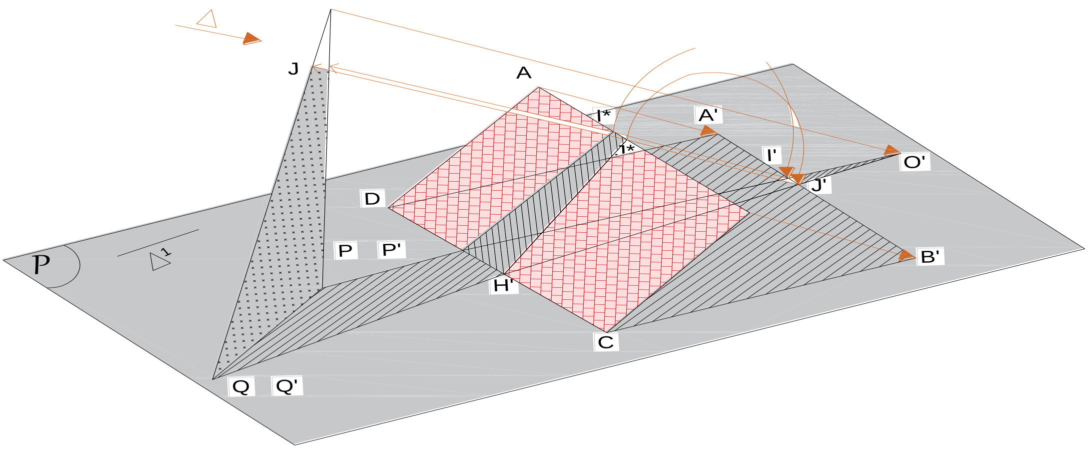


Figure 110 : source author

4 Projection of a Shadow of Point A (Application of the theory of shadow in monge projection):

To describe the orthogonal projection of the shadow of a point A in space:

Identify the Plane: Determine the plane onto which the point A will be projected. This could be any plane in space, commonly the xy-plane for simplicity.

Perpendicular Line: Draw a line from point A that is perpendicular to the chosen plane.

Intersection Point with the light source (45°): The point where this inclinar line intersects the plane is the projection of point A. This intersection point is the shadow of point A on the plane.

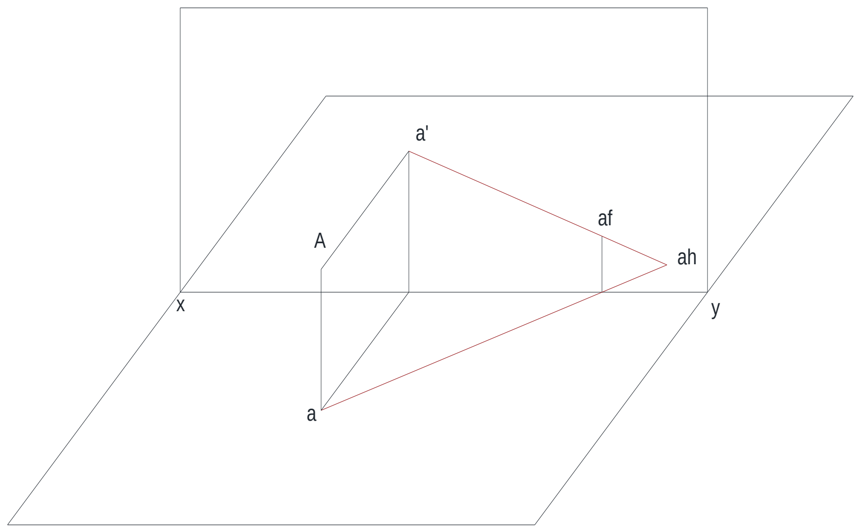


Figure 111: Application of the theory of shadows in monge projection, source author

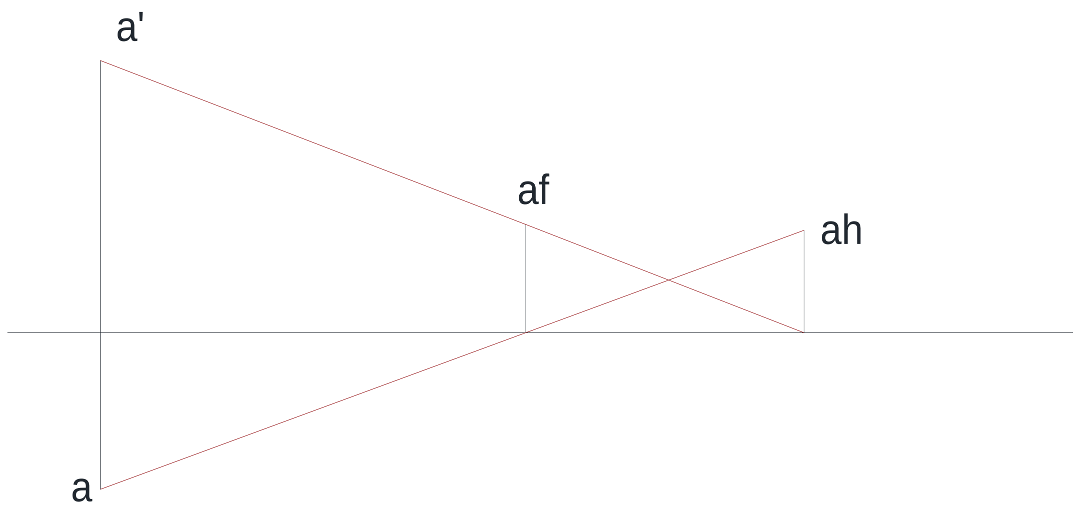


Figure 112 : source author

- If point A has coordinates (x, y, z) in 3D space, its orthogonal projection onto the xy-plane will be (x1, y1, 0). The z-coordinate is set to zero.

-Application :

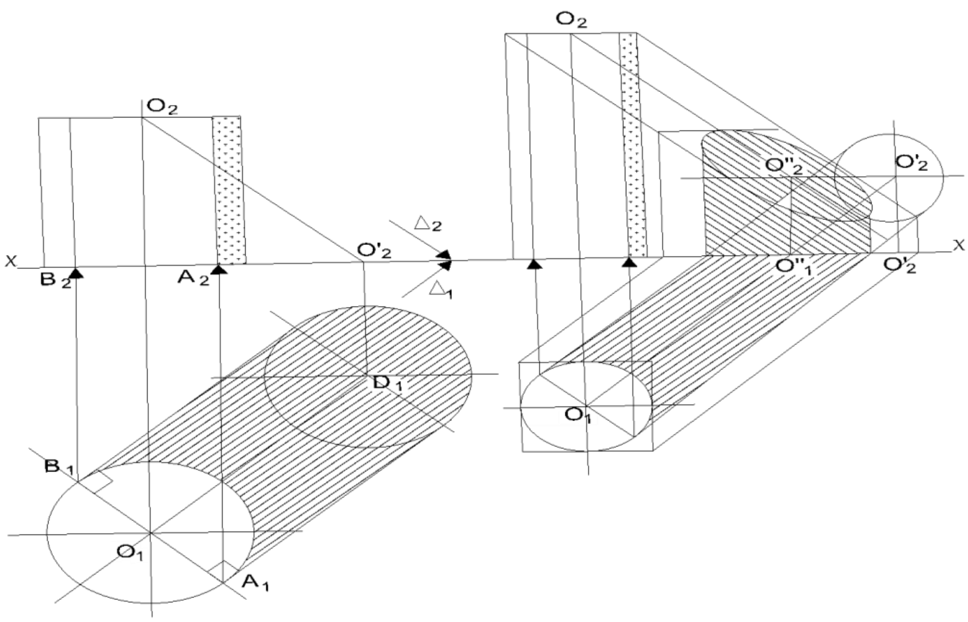


Figure 113: Application of the theory of shadows in monge projection,

source author

When a cylinder is illuminated by a light source, the shadow it casts on a plane depends on the orientation of the cylinder relative to the light source. In this case, the cylinder is placed on the ground, which means that the base is formed by points with coordinates z = 0, and the contour of the shadow develops directly from its horizontal projection to the projected shadow of the top of the object.

Top View: If the cylinder is positioned such that the light source is directly above it, the shadow will be a circle on the horizontal plane or a elipse on the frontal plane of projection.

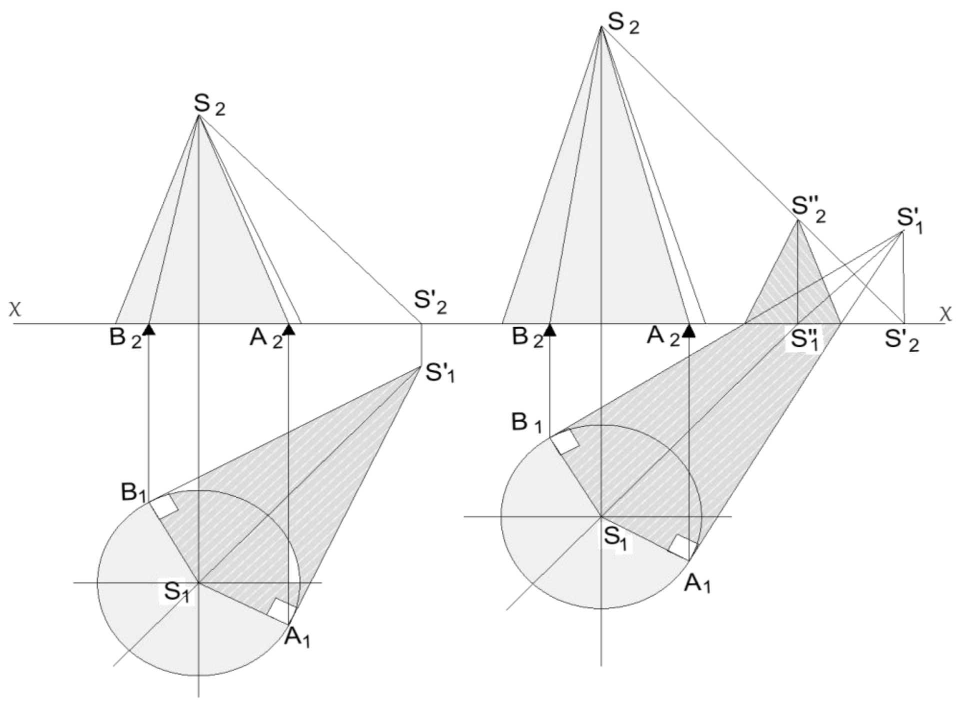


Figure 114 : source author

- Similar to the cylinder, the shadow of a cone also depends on its orientation relative to the light source. The top of the object in this situation is a point (three dimension) that will be connected to the base of the object when the shadow is on the horizontal plane. S1 determines the shadow of the top. In the second situation, we use the two projections of point S: s2 for tracing the shadow on the frontal plane and s1 to determine the inclination of the cone on the horizontal plane

- **"self-shadow" or "attached shadow" :** It refers to the shadow on the surface of the object itself, where the light does not reach.

- **"cast shadow" :** It refers to the shadow that an object projects onto another surface, resulting from the object blocking the light source.

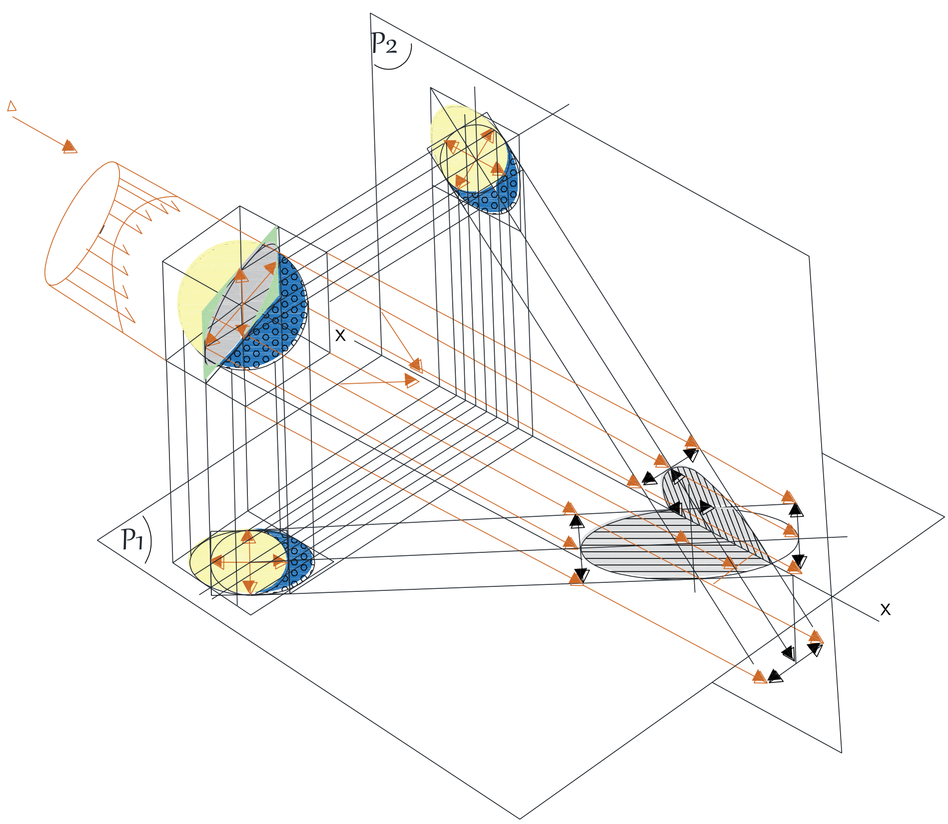


Figure 115 : **source author**

- **Applications :**

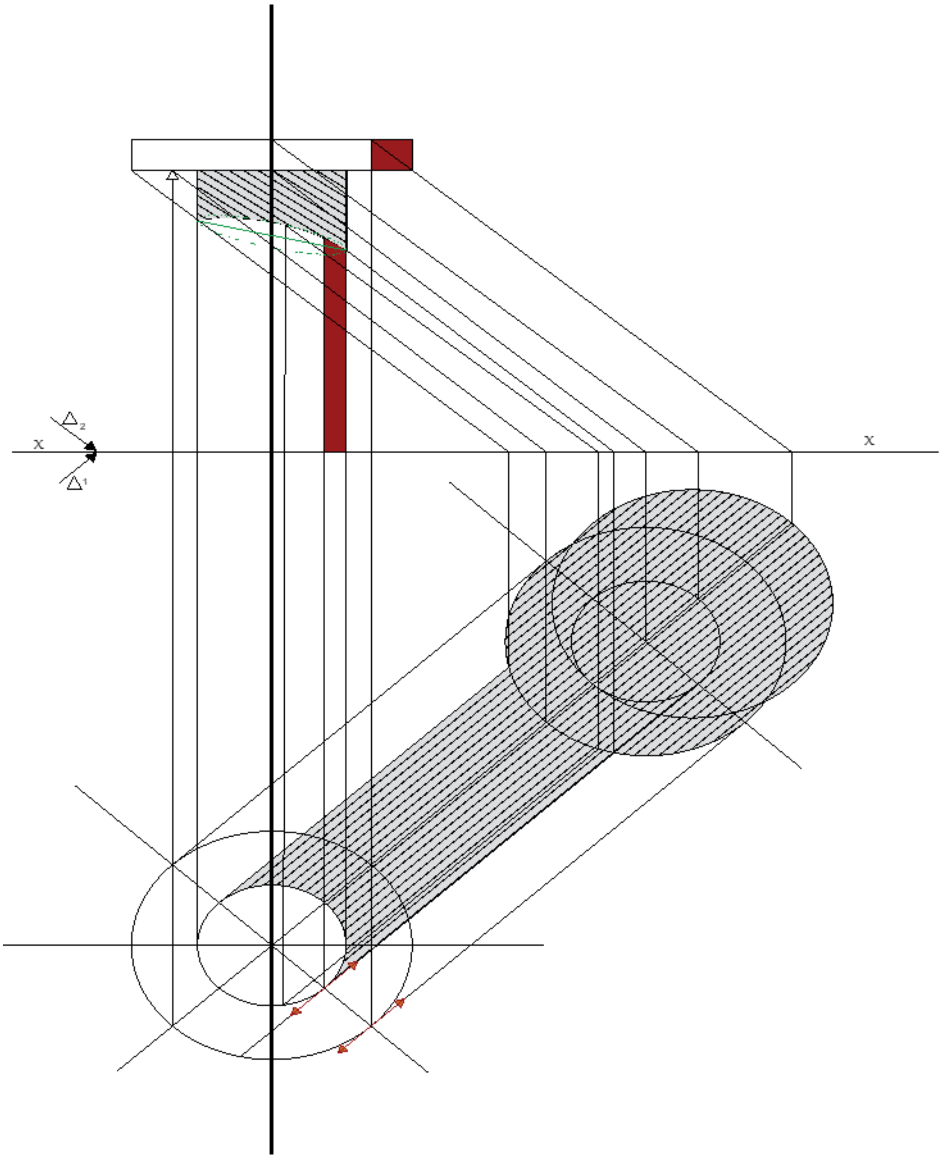


Figure 116 : **source author**

Here’s how this process works:

Light Source Interaction: When a light source illuminates a cylinder, parts of the cylinder's surface directly facing the light will be brightly lit, while other parts will receive less light or no direct light at all.

Shadow Formation (red color): The regions of the cylinder not facing (the middle of cylinder in this caces) the light source will fall into shadow. This shadow is termed a self-shadow because it is formed by the cylinder itself blocking the light.

5 Properties of Shadows :

- Dependence on Light Source, Object Shape and plane of projection :

The shape and position of the shadow are influenced by the shape of the object and the location of the light source and the projections plans.

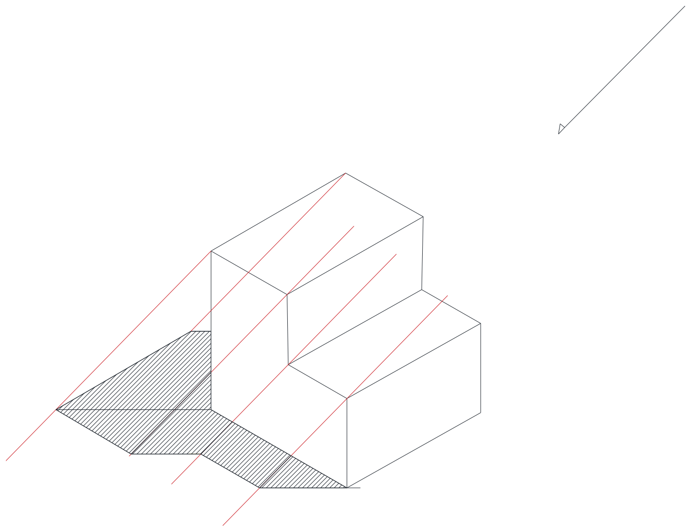


Figure 117: Application of the theory of shadows in axonometry, source author

The use of shadow theory in axonometric drawing is crucial for accurately and clearly depicting three-dimensional objects on a two-dimensional surface. Key aspects include:

Shadow Creation: Shadows are created by drawing rays from a specific light source to the object. The intersections of these rays with the object and the drawing plane outline the shadows' edges.

Shadow Types: In axonometry, shadows are divided into cast shadows and self shadows. Cast shadows fall onto another surface, while self shadows are areas of the object that remain unlit by the light source.

Realism and Consistency: To enhance realism, shadows must align with the light source's position and follow axonometric perspective principles. This alignment helps clarify the object's shape and volume.

In essence, using shadow theory in axonometry produces technical drawings that not only depict the objects' sizes and shapes but also illustrate how they interact with light, making the representations clearer and more lifelike.

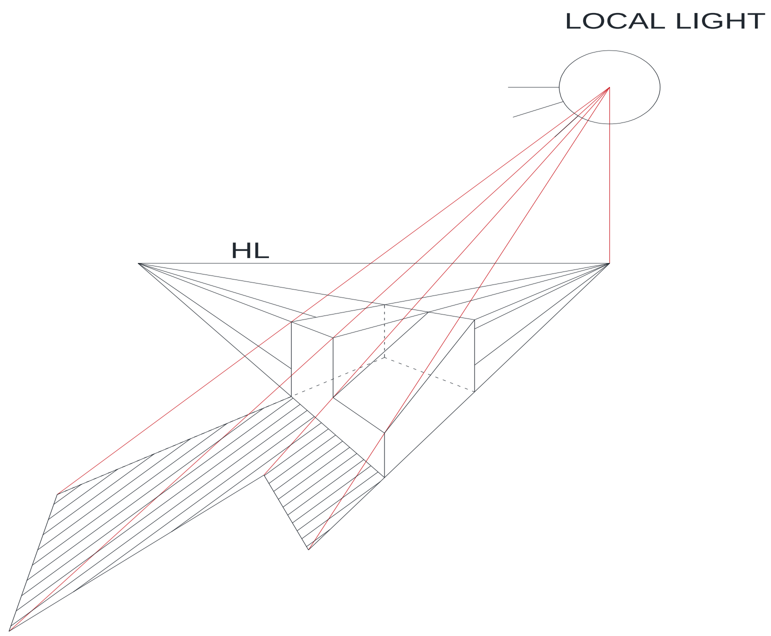


Figure 118 : Application of the theory of shadows in perspective, **source author**

Applying the theory of shadows in perspective is crucial for adding realism and depth to two-dimensional drawings in art and technical illustration. Key points include:

Shadow Casting: Determining shadow placement involves projecting lines from the light source to the object's edges. The points where these lines intersect the ground or other surfaces define the shadow's contour. This process requires careful consideration of the light source’s angle and position.

Shadow Types: Shadows in perspective include cast shadows and form shadows. Cast shadows are projected onto a surface by an object obstructing the light, while form shadows are the darker areas on the object itself where light does not reach.

Achieving Realism: Accurate shadows add depth and solidity to drawings, helping viewers perceive spatial relationships and enhancing the depiction's lifelike quality.

Art and Design Applications: Shadow theory is used in various fields such as fine art, architecture, and computer graphics. Shadows enhance visual narratives, create mood, and emphasize three-dimensional forms.

By mastering shadow theory in perspective, artists and designers can create more dynamic and realistic images, enriching the viewer’s perception and engagement with the artwork.

- **Application :**

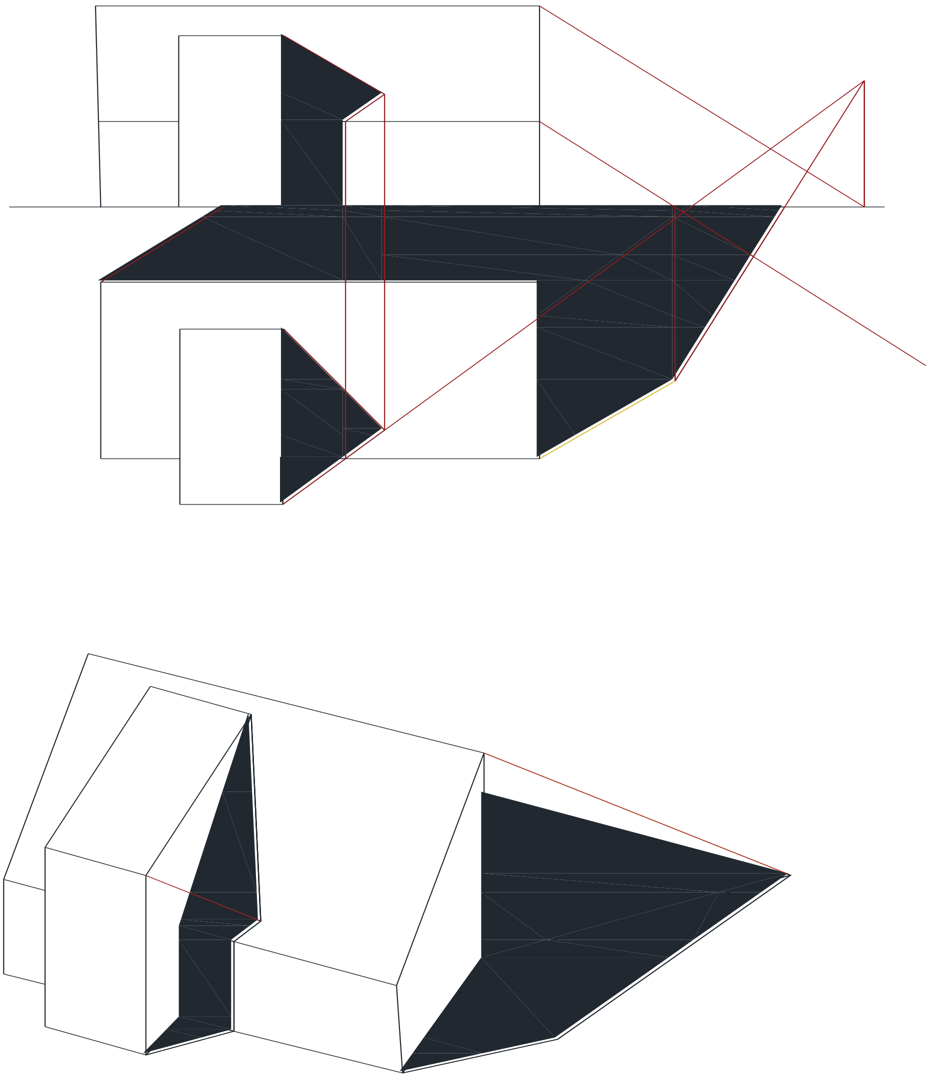


Figure 119 : source author

- Opaque Objects :

Shadows are cast only by opaque objects, which do not allow light to pass through.

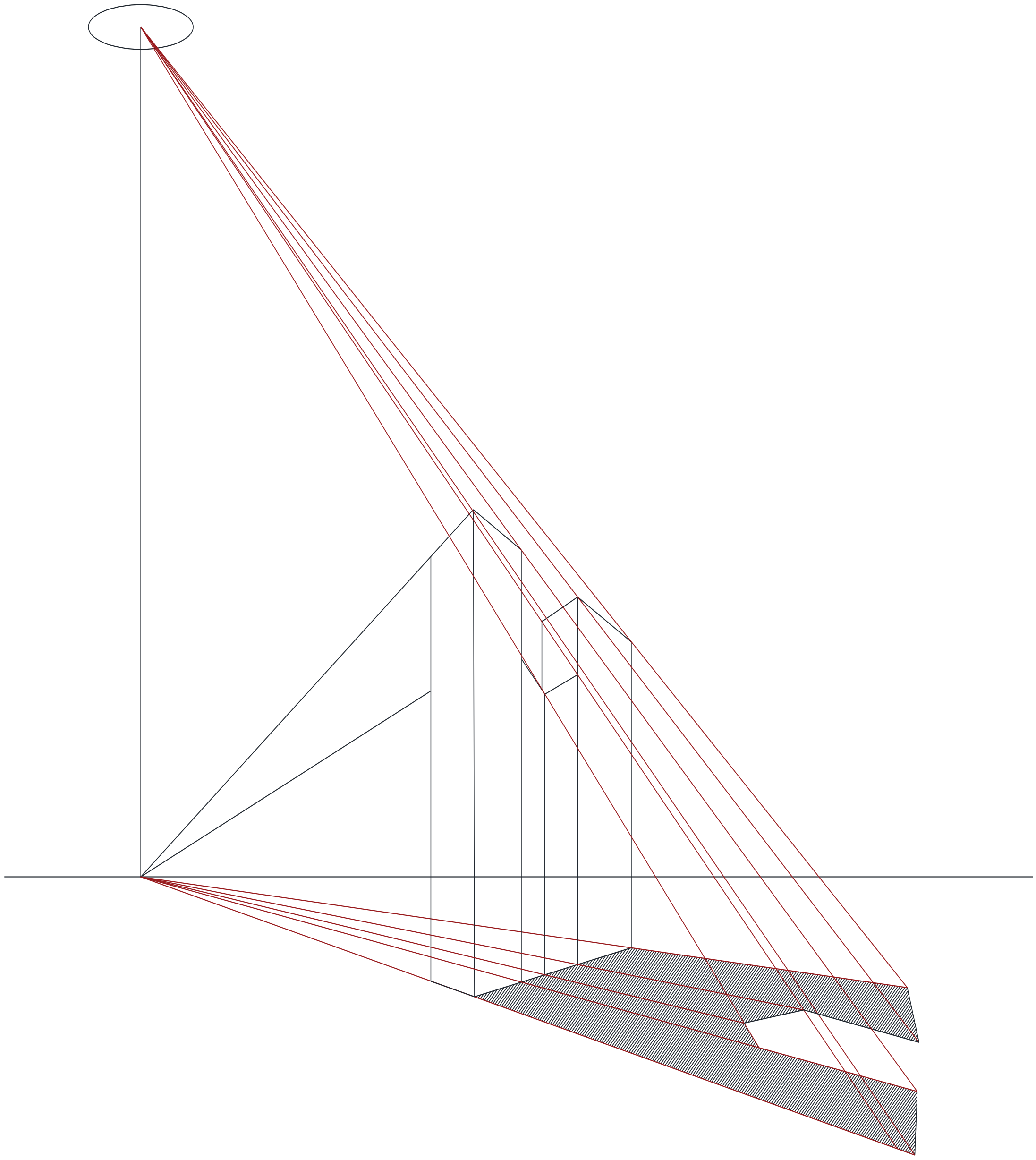


Figure 120 : source author

- Shadow Contours

The contours of a shadow are determined by the geometry of the object and the light source. This includes aspects such as the angles and distances involved.

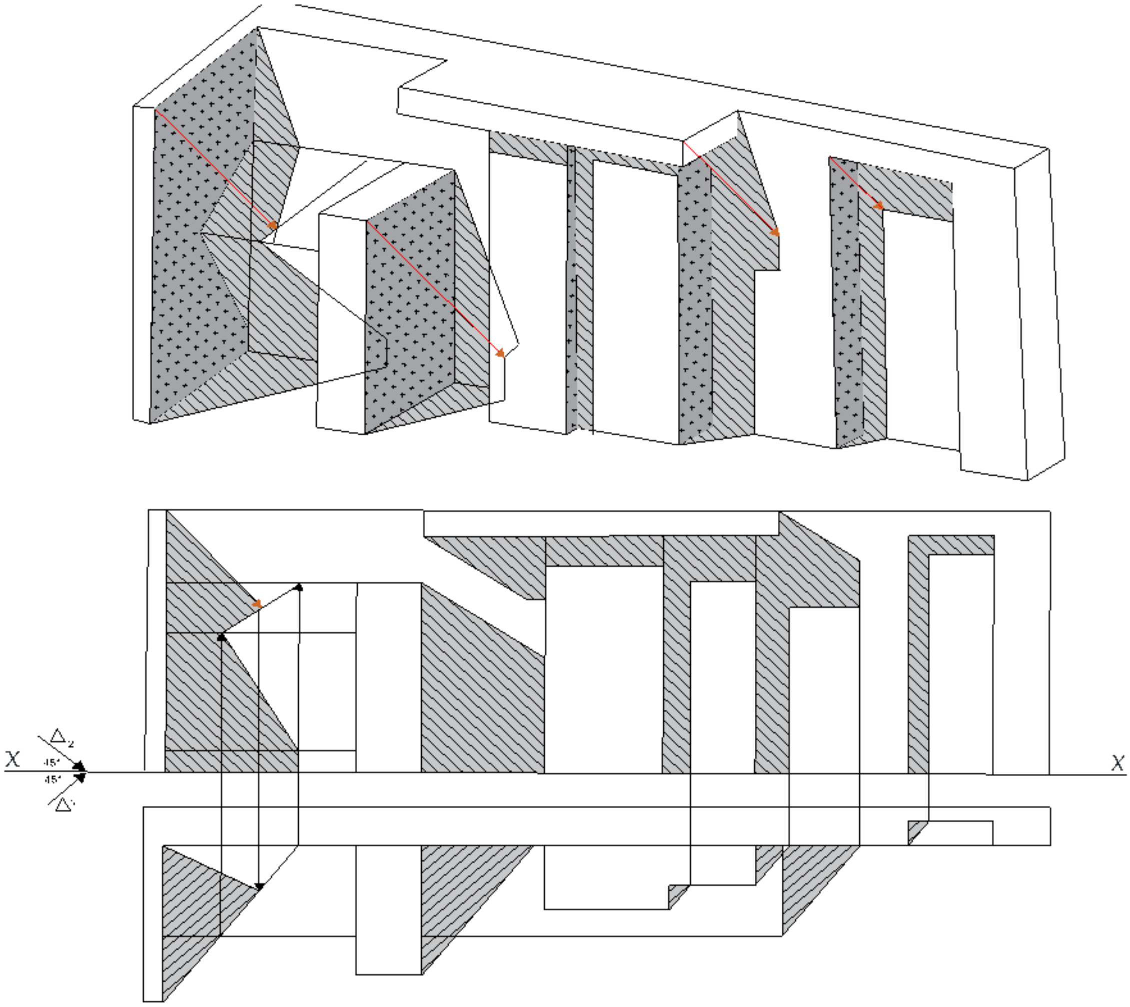


Figure 121 : **source author**

- Size and Position Variability

The size and position of a shadow can vary depending on the angle and distance of the light source relative to the object. For example, a shadow becomes longer as the light source is moved lower and shorter when the light source is higher.

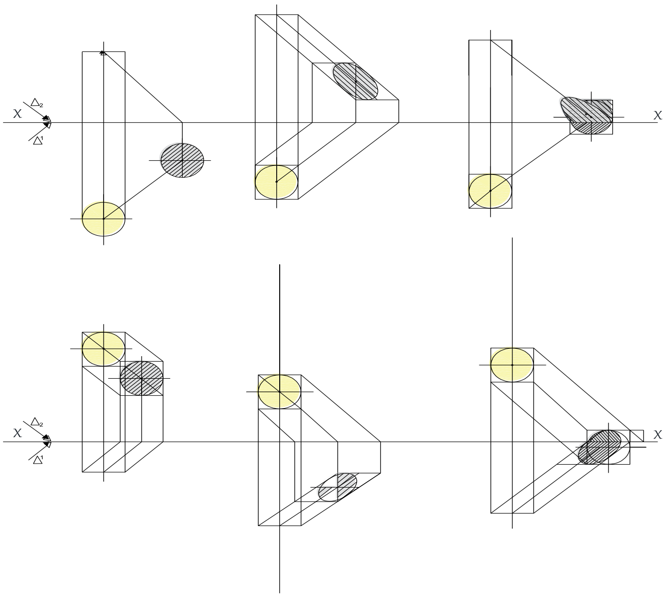


Figure 122 : source author

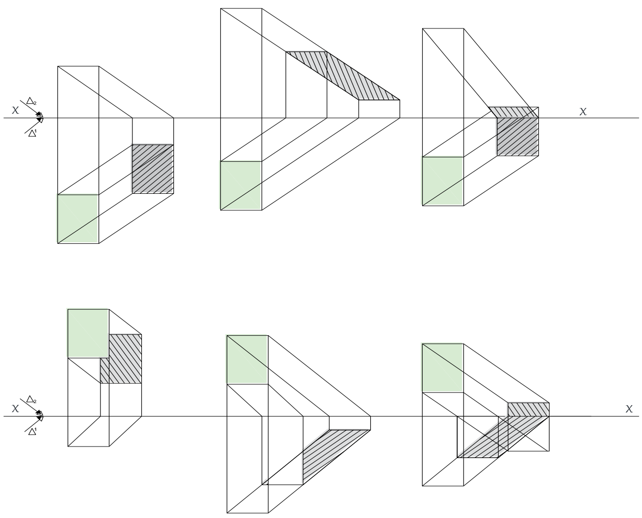


Figure 123 : source author

- Absence of Detail

Shadows typically do not reveal any details of the object except its outline or shape. No internal features of the object are visible in its shadow

Exercice :

1. What are the fundamental principles of Monge projection, and how do they apply to the theory of shadows?
2. How does the representation of shadows differ between Monge projection and axonometry?
3. What are the steps to construct shadows in a perspective drawing, and how do these steps ensure accurate representation of depth and light?
4. Can you describe a practical example where Monge projection is used to solve a shadow problem in architectural design?
5. What are the advantages and limitations of using axonometry for the application of the theory of shadows in technical drawings?
6. How does the angle of light affect the shadows in Monge projection compared to perspective drawing?
7. What role do orthographic projections play in understanding and applying the theory of shadows in descriptive geometry?
8. Describe the process of determining the shadow of a complex 3D object using Monge projection.

Bibliography:

1. "Descriptive Geometry" by V. O. Gordon
2. "A Course In Descriptive Geometry: For The Use In Colleges And Scientific Schools" by J.B. Lovell
3. "History of Descriptive Geometry with an Emphasis on Its Application in Engineering" by V. Moravcová
4. "Preparation of Descriptive Geometry Course in English" by B. Kotarska-Lewandowska
5. Le Dessin d'architecture à main levée » Magali Degado Yanes, Ernest Redondo Dominguez, Edition Eyrolles, Espagne 2007
6. « Dessin technique »Bert Bielefeld, Isabella Skiba, Editions BIRKHÄUSER
7. « Savoir faire de l'architecture ». « AXONOMETRIE ». « Théorie, art et pratique des perspectives parallèles : axonométrie orthogonale, axonométrie oblique, perspectives cavalière et militaire ». Jean Aubert, Editions de la Villette, Paris, 1996
8. « Perspectives coniques et axonométriques pas à pas » Gérard Calvat , Editions Eyrolles, 2000
9. « La perspective « pas à pas ». « Manuel de construction graphique de l'espace et tracé des ombres » Jean-Claude Ludi , DUNOD, Paris, 1999
10. « Perspective scientifique et artistique » Bernard Bodson, Editions Eyrolles
11. https://drawing-portal.com/en/zadachi-nachertatelnoj-geometrii.html : Written by Евгений. Posted in The solution of problems in descriptive geometry in Autocad

12. La double projection a l'usage du dessin d'architecture, Bachir Ribouh, office des publications universitaires.

13. Le dessin d'architecture, l'essentiel d'une mise en perspective, Tebibel Hadi, Bachir Ribouh, Bensakhria Karima. Office des publications universitaires.